

Implication of a vanishing element in 3+1 Scenario

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Abstract

In this paper we study the one zero textures of low energy neutrino mass matrices in presence of a sterile neutrino. We find that the mass matrix elements $m_{\alpha\beta}$ ($\alpha, \beta = e, \mu, \tau$) involving only the active states can assume vanishing values in the allowed parameter space for all the mass spectrum. Among the mass matrix elements connecting the active and sterile states, m_{es} and $m_{\mu s}$ can become small only for the quasi-degenerate neutrinos. The element $m_{\tau s}$ on the other hand can be very small even for lower values of masses since the 3-4 mixing angle only has an upper bound from current data. The mass matrix element (m_{ss}) involving only the sterile state stays $\sim \mathcal{O}(1)$ in the whole parameter region. We study the possible correlations between the sterile mixing angles and the Majorana phases to give a zero element in the mass matrix.

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I. INTRODUCTION

Light sterile neutrinos were invoked to explain the results of the LSND experiment which reported oscillation events in the $\bar{\nu}_\mu - \bar{\nu}_e$ mode corresponding to a mass squared difference $\sim \text{eV}^2$ [1]. Adding one sterile neutrino to the standard 3 generation framework gives rise to two possible mass spectra – the 2+2 in which two pairs of mass states are separated by a difference $\sim \text{eV}^2$ and 3+1 in which a single predominantly sterile state differs by $\sim \text{eV}^2$ from the three active states [2]. Subsequently the 2+2 schemes were found to be incompatible with the solar and atmospheric neutrino data [3]. The MiniBoone experiment was designed to test this and its antineutrino data confirmed the LSND anomaly [4]. Both 3+1 and 3+2 sterile neutrino schemes have been considered to explain these results [5–7]. Such global fits aim to explain the non-observance of eV^2 oscillations in the disappearance channel in other short baseline experiments as well as the reported evidence in LSND/ MiniBoone experiments. The relevant probabilities for 3+1 case is governed by a single mass squared difference and hence is independent of the CP phase. In the 3+2 scheme dependence on CP phase comes into play and one gets a slightly better fit.

Other evidences in support of sterile neutrinos include – the reactor and Ga anomaly. The first one refers to the deficit in the measured electron antineutrino flux in several experiments when the theoretical predictions of reactor neutrino fluxes were reevaluated [8]. The second one implies shortfall of electron neutrinos observed in the solar neutrino detectors GALLEX and SAGE using radioactive sources [9]. Both these can be explained by adding light sub-eV sterile neutrinos in the three generation framework.

There has also been some hint in favour of sterile neutrinos from cosmological observations of a "dark radiation" which is weakly interacting and relativistic. Attributing this to sterile neutrinos one gets the bound on the number of neutrinos as $N_{eff} = 4.34 \pm 0.87$ at 68% C.L [10]. The Plank satellite experiment which has very recently declared its first results [11], on the other hand, gives $N_{eff} = 3.30 \pm 0.27$ at 68% C.L. which allows for an extra sterile neutrino at 95% C.L., although its mixing with active species can be very tightly constrained [12] within the framework of standard cosmology. Thus the sterile neutrinos continue to be intriguing and many new experiments are planned/proposed to test this [13].

Theoretically, sterile neutrinos are naturally included in Type-I seesaw model [14]. But their mass scale is usually very high to account for the small mass of the neutrinos. Light

sub-eV sterile neutrinos as suggested by the data can arise in many models [13].

Irrespective of the mechanism for generation of neutrino masses the low energy Majorana mass matrix in presence of an extra sterile neutrino will be of dimension 4×4 with ten independent entries and is given as,

$$M_\nu = V^* M_\nu^{diag} V^\dagger \quad (1)$$

where, $M_\nu^{diag} = \text{Diag}(m_1, m_2, m_3, m_4)$ and V denotes the leptonic mixing matrix in a basis where the charged lepton mass matrix is diagonal. One of the important aspects in the study of neutrino physics is to explore the structure of the neutrino mass matrices. At the fundamental level these are governed by Yukawa couplings which are essentially free parameters in most models. These motivated the study of texture zeros which means one or more elements are relatively small compared to the others. Texture zeros in the low energy mass matrices in the context of three generations have been extensively explored both in the quark and lepton sector [15], [16]. Such studies help in understanding the underlying parameter space and the nature of the mass spectrum involved and often predict correlations between various parameters which can be experimentally tested. For three generation scenario it is well known that the number of maximum texture zeros in low energy mass matrix is two [15]. In the context of the 4-neutrino case however more than two zeros can be allowed [17]. Two zero textures of sterile neutrinos have been studied recently in [17] and three zero cases have been considered in [18]. In this paper we concentrate on the textures where one of the mass matrix elements is vanishing. For the 4×4 symmetric mass matrix it gives total 10 different cases which needs to be investigated. We study the implications of one zero textures and the possible correlations between the parameters. We also compare our results with the 1 zero textures for three active neutrinos [19, 20].

The plan of the paper goes as follows. In section II we discuss the possible mass spectra and the mixing matrix in the 3+1 scenario. In the next section we present our study regarding the implications of one vanishing entry in the low energy neutrino mass matrix. We conclude in section IV.

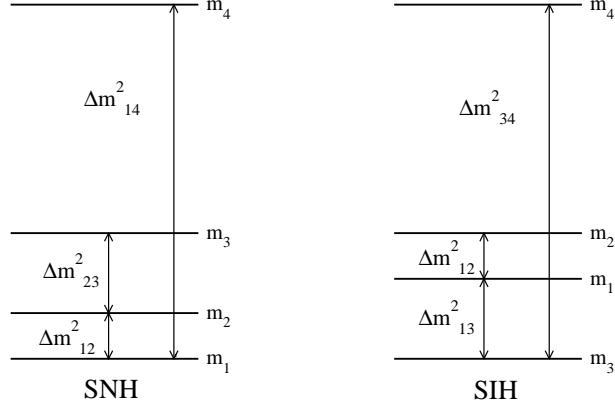


Figure 1: The allowed 3+1 mass ordering

II. MASSES AND MIXING IN THE 3+1 SCHEME

There are two ways in which one can add a predominantly sterile state separated by $\sim \text{eV}^2$ from the standard 3 neutrino mass states. In the first case the additional neutrino can be of higher mass than the other three while in the second case the the fourth neutrino is the lightest state. The later turns out to be incompatible with cosmology since in this case three active neutrinos, each with mass $\sim \text{eV}$ results in an enhanced cosmological energy density. Thus it suffices to consider only the first case which admits two possibilities displayed in Fig. 1.

- (i) SNH: in this $m_1 \approx m_2 < m_3 < m_4$ corresponding to a normal hierarchy (NH) among the active neutrinos which implies,

$$m_2 = \sqrt{m_1^2 + \Delta m_{12}^2}, m_3 = \sqrt{m_1^2 + \Delta m_{12}^2 + \Delta m_{23}^2}, m_4 = \sqrt{m_1^2 + \Delta m_{14}^2}.$$

- (ii) SIH : this corresponds to $m_3 < m_2 \approx m_1 < m_4$ implying an inverted ordering among the active neutrinos with masses expressed as,

$$m_1 = \sqrt{m_3^2 + \Delta m_{13}^2}, m_2 = \sqrt{m_3^2 + \Delta m_{13}^2 + \Delta m_{12}^2}, m_4 = \sqrt{m_3^2 + \Delta m_{34}^2}.$$

Here, $\Delta m_{ij}^2 = m_j^2 - m_i^2$. In the extreme cases these masses can be written as

$$SNH : |m_4| \approx \sqrt{\Delta m_{14}^2} \gg |m_3| \approx \sqrt{\Delta m_{23}^2} \gg |m_2| \approx \sqrt{\Delta m_{12}^2} \gg |m_1|, \quad (2)$$

$$SIH : |m_4| \approx \sqrt{\Delta m_{34}^2} \gg |m_2| \approx |m_1| \approx \sqrt{\Delta m_{23}^2} \gg |m_3|, \quad (3)$$

$$SQD : |m_4| \gg |m_1| \approx |m_2| \approx |m_3| \approx m_0. \quad (4)$$

The first two cases correspond to complete hierarchy among the active neutrinos while the last one is the quasi-degenerate (QD) regime where the three active neutrinos have approximately equal masses.

In the 3+1 scenario, the neutrino mixing matrix, V in the flavor basis can be parametrized in terms of sixteen parameters. In addition to the three mixing angles between the active flavors, $(\theta_{13}, \theta_{12}, \theta_{23})$ we now have three more mixing angles from sterile and active mixing, $(\theta_{14}, \theta_{24}, \theta_{34})$. There are six CP violating phases, three Dirac $(\delta_{13}, \delta_{14}, \delta_{24})$ and three additional Majorana phases as (α, β, γ) as neutrinos here are considered to be Majorana particles. Then, there are four masses of neutrino m_1, m_2, m_3 corresponding to three active states and m_4 which is predominantly the mass of heavy sterile neutrino.

The mixing matrix V can be expressed as $V = U.P$ [21] where

$$U = R_{34}\tilde{R}_{24}\tilde{R}_{14}R_{23}\tilde{R}_{13}R_{12} \quad (5)$$

where R_{ij} denotes rotation matrices in the ij generation space and is expressed as,

$$R_{34} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & c_{34} & s_{34} \\ 0 & 0 & -s_{34} & c_{34} \end{pmatrix}, \quad \tilde{R}_{14} = \begin{pmatrix} c_{14} & 0 & 0 & s_{14}e^{-i\delta_{14}} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s_{14}e^{i\delta_{14}} & 0 & 0 & c_{14} \end{pmatrix}$$

Here we use the abbreviations $s_{ij} = \sin \theta_{ij}$ and $c_{ij} = \cos \theta_{ij}$. The phase matrix is diagonal and is expressed as,

$$P = \text{Diag}(1, e^{i\alpha}, e^{i(\beta+\delta_{13})}, e^{i(\gamma+\delta_{14})}).$$

| Parameter | Best Fit values | 3σ range |
|--|-----------------|-----------------|
| $\Delta m_{12}^2/10^{-5} \text{ eV}^2$ (NH or IH) | 7.54 | 6.99 – 8.18 |
| $\sin^2 \theta_{12}/10^{-1}$ (NH or IH) | 3.07 | 2.59 – 3.59 |
| $\Delta m_{23}^2/10^{-3} \text{ eV}^2$ (NH) | 2.43 | 2.19 – 2.62 |
| $\Delta m_{13}^2/10^{-3} \text{ eV}^2$ (IH) | 2.42 | 2.17 – 2.61 |
| $\sin^2 \theta_{13}/10^{-2}$ (NH) | 2.41 | 1.69 – 3.13 |
| $\sin^2 \theta_{13}/10^{-2}$ (IH) | 2.44 | 1.71 – 3.15 |
| $\sin^2 \theta_{23}/10^{-1}$ (NH) | 3.86 | 3.31 – 6.37 |
| $\sin^2 \theta_{23}/10^{-1}$ (IH) | 3.92 | 3.35 – 6.63 |
| $\Delta m_{LSD}^2(\Delta m_{14}^2 \text{ or } \Delta m_{34}^2) \text{ eV}^2$ | 1.62 | 0.7 – 2.5 |
| $\sin^2 \theta_{14}$ | 0.03 | 0.01 – 0.06 |
| $\sin^2 \theta_{24}$ | 0.01 | 0.002 – 0.04 |
| $\sin^2 \theta_{34}$ | – | < 0.18 |
| $\zeta/10^{-2}$ (NH) | – | 2.7 – 3.7 |
| $\zeta/10^{-2}$ (IH) | – | 2.7 – 3.8 |
| $\xi/10^3$ (NH) | – | 0.27–1.14 |
| $\xi/10^3$ (IH) | – | 0.27– 1.15 |

TABLE I: 3σ ranges of neutrino oscillation parameters [22]. The current constraints on sterile neutrino parameters are from [23], [24]. Also given are the 3σ ranges of the mass ratios $\zeta = \frac{\Delta m_{12}^2}{|\Delta m_{23}^2|}$ and $\xi = \frac{\Delta m_{LSD}^2}{|\Delta m_{23}^2|}$.

The best-fit values and the 3σ ranges of the oscillation parameters in the 3+1 scenario are given in Table I where in addition to the masses and mixing angles we also present the mass ratios $\zeta = \frac{\Delta m_{12}^2}{|\Delta m_{23}^2|}$ and $\xi = \frac{\Delta m_{LSD}^2}{|\Delta m_{23}^2|}$ which would be useful in our analysis.

In Fig. 2 we have plotted the sum of neutrino masses against the lowest neutrino mass for both NH and IH. The band corresponds to variation of the mass squared differences in their current 3σ range. We also show the cosmological upper bound on neutrino masses in

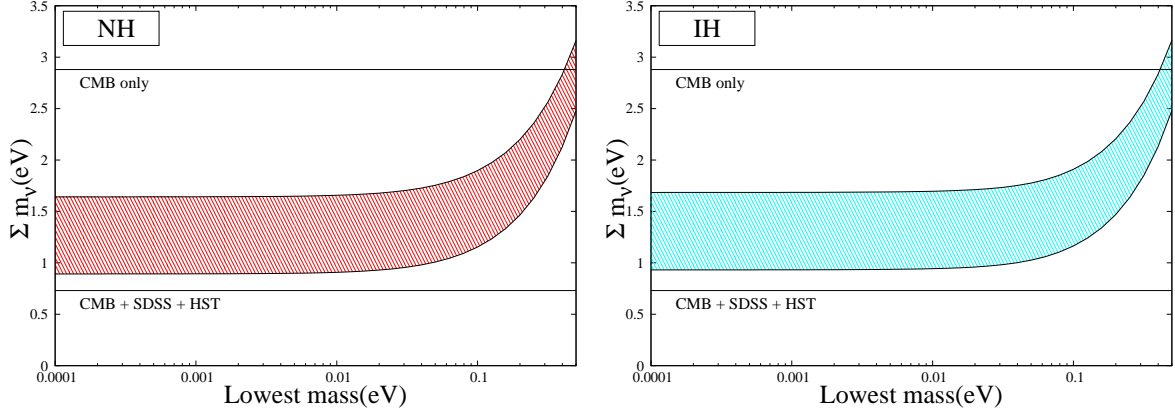


Figure 2: Plots of sum of light neutrino masses (Σm_ν) vs the lowest mass in the 3+1 scenario. Also shown are the cosmological upper bound on neutrino mass from the analysis of CMB data plus matter power spectrum information (SDSS) and a prior on H_0 (HST) and from the analysis of CMB data only from reference [23] for 3+1 scheme.

3+1 scenario from [23]. The combined analysis of CMB + SDSS + HST seems to rule out the mass spectrum of 3+1 scenario whereas if only CMB data is taken then region for the lowest mass < 0.4 eV gets allowed for both the hierarchies. Note that the analysis in [23] does not incorporate the Planck results [11] which can constrain the sum of masses further, In our analysis we have varied the lowest mass up to 0.5 eV.

III. NEUTRINO MASS MATRIX ELEMENTS

In this section we study the implication of the condition of vanishing $m_{\alpha\beta}$ for the 3+1 scenario, where $\alpha, \beta = e, \mu, \tau, s$. Since $m_{\alpha\beta}$ is complex the above condition implies both real and imaginary parts are zero. Therefore to study the 1-zero textures we consider $|m_{\alpha\beta}| = 0$.

A. The Mass Matrix element m_{ee}

The matrix element m_{ee} in the 3+1 scenario is given as,

$$m_{ee} = m_1 c_{14}^2 c_{13}^2 c_{12}^2 + m_2 s_{12}^2 c_{14}^2 c_{13}^2 e^{2i\alpha} + m_3 s_{13}^2 c_{14}^2 e^{2i\beta} + m_4 s_{14}^2 e^{2i\gamma}. \quad (6)$$

This is of the form

$$m_{ee} = c_{14}^2 (m_{ee})_{3\nu} + e^{2i\gamma} s_{14}^2 m_4, \quad (7)$$

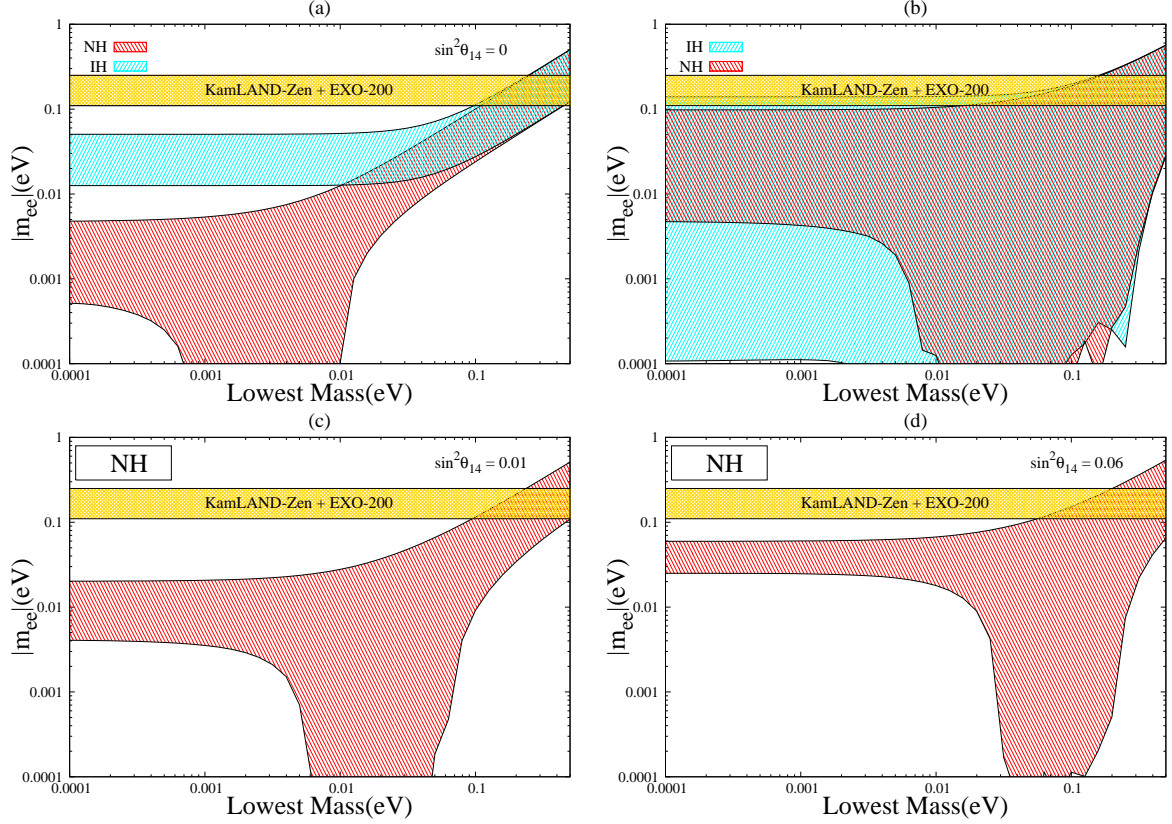


Figure 3: Plot of $|m_{ee}|$ versus the lowest mass. The panel (a) corresponds to the three generation case while panel (b) is for 3+1 case. In panel (b) all the mixing angles are varied in their 3σ range and the CP violating phases are varied in their full range. The panel (c) and (d) are for specific values of θ_{14} with all other parameters covering their full range.

where $(m_{ee})_{3\nu}$ corresponds to the matrix element in the 3 active neutrino case. The contribution of the sterile neutrino to the element m_{ee} depends on the mass m_4 and the active-sterile mixing angle θ_{14} . Of all the mass matrix element m_{ee} has the simplest form because of the chosen parametrization and can be understood quite well. Using approximation in Eq. (2) for the case of extreme hierarchy one can write this for NH as,

$$m_{ee} \approx c_{14}^2 (m_{ee})_{3\nu} + e^{2i\gamma} s_{14}^2 \sqrt{\Delta m_{14}^2}, \quad (8)$$

where $(m_{ee})_{3\nu} \approx \sqrt{\Delta m_{23}^2} (e^{2i\alpha} c_{13}^2 s_{12}^2 \sqrt{\zeta} + s_{13}^2 e^{2i\beta})$ and, $\zeta = \frac{\Delta m_{12}^2}{|\Delta m_{23}^2|}$ is the ratio of the solar and the atmospheric mass squared differences. The modulus of m_{ee} is the effective mass that can be extracted from half life measurements in neutrinoless double beta decay. In Fig. 3 we plot the effective mass as a function of the smallest mass by varying θ_{14} in its complete 3σ range from Table I as well as for specific values of the mixing angle θ_{14} . The

Majorana phases are varied randomly in the range 0 to π in all the plots. The first panel is for $\theta_{14} = 0$ i.e the three generation case. It is seen that for present values of the oscillation parameters the cancellation condition is not satisfied for $m_1 \rightarrow 0$ for NH. However, as one increases m_1 , complete cancellation can be achieved. For IH the complete cancellation is never possible. These results change when we include the sterile contribution as is evident from the panel (b) in Fig. 3 which shows the effective mass for NH and IH by varying all the parameters in their full 3σ allowed range. The behaviour can be understood from the expressions of $|m_{ee}|$ in various limiting cases. For NH, in the hierarchical limit of $m_1 \rightarrow 0$ the major contributor will be the additional term due to the sterile neutrinos because of higher value of m_4 . Complete cancellation is only possible for smaller values of θ_{14} so that this contribution is suppressed. The typical value of θ_{14} required for cancellation can be obtained by putting $\alpha = \beta = 0$ (which would maximize the three neutrino contribution) and $\gamma = \pi/2$, as

$$\tan^2 \theta_{14} \approx \frac{(\sqrt{\xi} c_{13}^2 s_{12}^2 + s_{13}^2)}{\sqrt{\xi}} \approx 10^{-3}, \quad (9)$$

which lies outside the allowed range of θ_{14} given in Table I. Here $\xi = \frac{\Delta m_{14}^2}{|\Delta m_{23}^2|}$. As we increase m_1 , $(m_{ee})_{3\nu}$ increases and can be of the same order of magnitude of the sterile term. Hence one can get cancellation regions. The cancellation is mainly controlled by the value of θ_{14} . For higher values of s_{14}^2 one needs a higher value of m_1 for cancellation to occur. The correlation between m_1 and θ_{14} is brought out by the panels (c) and (d) in Fig. 3.

For IH case, in the limit of vanishing m_3 using approximation in Eq. (3), m_{ee} in a 3+1 scenario can be written as

$$|m_{ee}| \approx |c_{14}^2 c_{13}^2 \sqrt{\Delta m_{23}^2} (c_{12}^2 + s_{12}^2 e^{2i\alpha}) + \sqrt{\Delta m_{34}^2} s_{14}^2 e^{2i\gamma}|. \quad (10)$$

The maximum value of this is achieved for $\alpha = \gamma = 0$ which is slightly lower than that of NH in this limit. The element vanishes in the limit $m_3 \approx 0$ eV when $\alpha = 0$ and $\gamma = \pi/2$ provided

$$\tan^2 \theta_{14} \approx \frac{c_{13}^2}{\sqrt{\xi}} \approx 0.05 \quad (11)$$

This is well within the allowed range. This behaviour is in stark contrast to that in the 3 neutrino case [25]. There is no significant change in this behaviour as the smallest mass m_3 is increased since this contribution is suppressed by the s_{13}^2 term and the dominant contribution to $(m_{ee})_{3\nu}$ comes from the first two terms in Eq. (6). Therefore in this case we do not observe any correlation between m_3 and s_{14}^2 .

While moving towards the quasi-degenerate regime of $m_1 \approx m_2 \approx m_3$ we find that effective mass can still be zero. However, when the lightest mass approaches a larger value ~ 0.3 eV we need very large values of active sterile mixing angle θ_{14} , outside the allowed range, for cancellation. Hence the effective mass cannot vanish for such values of masses.

Also shown is the current limit on effective mass from combined KamLAND-Zen and EXO 200 results on the half-life of $0\nu\beta\beta$ in ^{136}Xe [26, 27]. When translated in terms of effective mass this corresponds to the bound $|m_{ee}| < 0.11 - 0.24$ eV including nuclear matrix element uncertainties. For the three generation case, the hierarchical neutrinos cannot saturate this bound. But in the 3+1 scenario this bound can be reached even for very small values of m_3 for IH and for some parameter values it can even exceed the current limit. Thus from the present limits on neutrinoless double beta decay searches a part of the parameter space for smaller values of m_3 can be disfavoured for IH. For NH, the KamLAND-Zen + EXO 200 combined bound is reached for $m_1 = 0.02$ eV and again some part of the parameter space can be disfavoured by this bound.

B. The Mass Matrix element $m_{e\mu}$

The mass matrix element $m_{e\mu}$ in the presence of extra sterile neutrino is given as

$$\begin{aligned} m_{e\mu} = & c_{14}(e^{i(\delta_{14}-\delta_{24}+2\gamma)}m_4s_{14}s_{24} + e^{i(\delta_{13}+2\beta)}m_3s_{13}(c_{13}c_{24}s_{23} - e^{i(\delta_{14}-\delta_{13}-\delta_{24})} \\ & s_{13}s_{14}s_{24}) + c_{12}c_{13}m_1(-c_{23}c_{24}s_{12} + c_{12}(-e^{i\delta_{13}}c_{24}s_{13}s_{23} - e^{i(\delta_{14}-\delta_{24})}c_{13}s_{14}s_{24})) \\ & + e^{2i\alpha}m_2c_{13}s_{12}(c_{12}c_{23}c_{24} + s_{12}(-e^{i\delta_{13}}c_{24}s_{13}s_{23} - e^{i(\delta_{14}-\delta_{24})}c_{13}s_{14}s_{24}))). \end{aligned} \quad (12)$$

Unlike m_{ee} here the expression is complicated and an analytic understanding is difficult from the full expression. The expression for $m_{e\mu}$ in the limit of vanishing active sterile mixing angle θ_{24} becomes

$$m_{e\mu} = c_{14}(m_{e\mu})_{3\nu}.$$

Since the active sterile mixing is small, in order to simplify these expressions we introduce a quantity $\lambda \equiv 0.2$ and define these small angles to be of the form $a\lambda$. Thus a systematic expansion in terms of λ can be done. For sterile mixing angle

$$\sin\theta_{14} \approx \theta_{14} \equiv \chi_{14}\lambda,$$

$$\sin\theta_{24} \approx \theta_{24} \equiv \chi_{24}\lambda, \quad (13)$$

and the reactor mixing angle as

$$\sin\theta_{13} \approx \theta_{13} \equiv \chi_{13}\lambda. \quad (14)$$

Here χ_{ij} are parameters of $\mathcal{O}(1)$ and their 3σ range from the current constraint on the mixing angles is given by

$$\chi_{13} = 0.65 - 0.9, \quad (15)$$

$$\chi_{14} = 0.5 - 1.2,$$

$$\chi_{24} = 0.25 - 1.$$

Using the approximation in Eqs. (2), (13) and (14) we get the expression for $|m_{e\mu}|$ for normal hierarchy as

$$\begin{aligned} |m_{e\mu}| \approx & \left| \sqrt{\Delta m_{23}^2} \{ \sqrt{\zeta} s_{12} c_{12} c_{23} e^{2i\alpha} + e^{i\delta_{13}} (e^{2i\beta} - e^{2i\alpha} \sqrt{\zeta} s_{12}^2) s_{23} \lambda \chi_{13} \right. \\ & \left. + \lambda^2 e^{i(\delta_{14} - \delta_{24})} (e^{2i\gamma} \sqrt{\xi} - e^{2i\alpha} \sqrt{\zeta} s_{12}^2) \chi_{14} \chi_{24} \} \right|. \end{aligned} \quad (16)$$

To see the order of magnitude of the different terms we choose vanishing Majorana phases while Dirac CP phases are taken as π . The mass matrix element $m_{e\mu}$ vanishes when

$$\sqrt{\zeta} s_{12} c_{12} c_{23} - (1 - \sqrt{\zeta} s_{12}^2) s_{23} \lambda \chi_{13} + \lambda^2 (\sqrt{\xi} - \sqrt{\zeta} s_{12}^2) \chi_{14} \chi_{24} = 0. \quad (17)$$

The three generation limit is recovered for $s_{24}^2 = 0$ and in panel (a) of Fig. 4 we show $|m_{e\mu}|$ as a function of m_1 of this case, for NH. Panel (b) (red/light region) of Fig. 4 shows $|m_{e\mu}|$ for the 3+1 case, with all parameters varied randomly within their 3σ range. The figures show that $|m_{e\mu}| = 0$ can be achieved over the whole range of the smallest mass for both 3 and 3+1 cases. However, we find that in the hierarchical limit cancellation is not achieved for large values of θ_{24} , since in that case the third term of Eq. (17) will be of the $\mathcal{O}(10^{-1})$ compared to the leading order term which is of the $\mathcal{O}(10^{-2})$ and hence there will be no cancellation of these terms. This can be seen from panel (b) (green/dark region) of Fig. 4 for $s_{24}^2 = 0.04$. In the QD limit the contribution from the active terms are large enough to cancel the sterile contribution and thus $|m_{e\mu}| = 0$ can be achieved.

For IH using the approximation Eq. (3) for the hierarchical limit we get the expression

$$|m_{e\mu}| \approx \left| \sqrt{\Delta m_{23}^2} \{ c_{12} s_{12} c_{23} (e^{2i\alpha} - 1) - e^{i\delta_{13}} (c_{12}^2 + s_{12}^2 e^{2i\alpha}) s_{23} \chi_{13} \lambda \right| \quad (18)$$

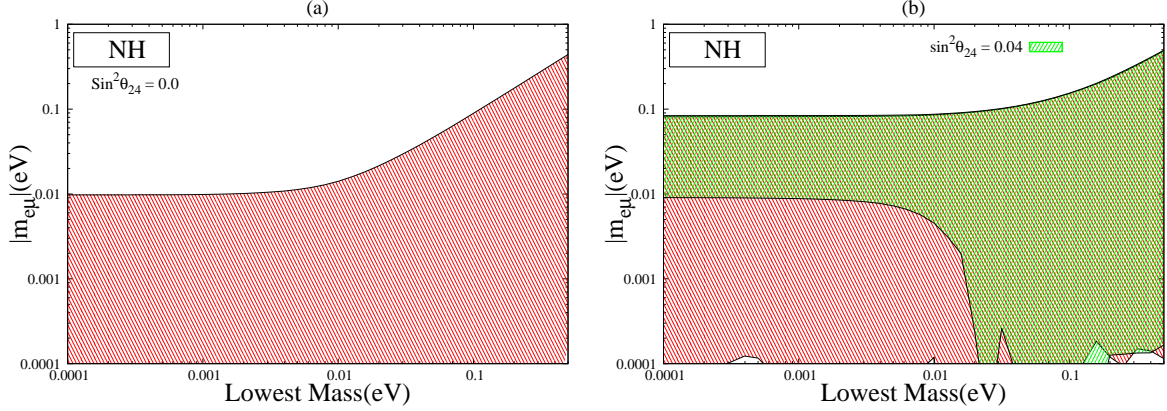


Figure 4: Plots of $|m_{e\mu}|$ as a function of the lowest mass m_1 for NH. Panel (a) correspond to the three generation case while (b) (red/light region) is for 3+1 case and also for $s_{24}^2 = 0.04$ (green/dark region). All the parameters are varied in their full 3σ allowed range and the CP violating phases are varied from 0 to π unless otherwise stated.

$$- e^{i(\delta_{14}-\delta_{24})} \lambda^2 \chi_{14} \chi_{24} (c_{12}^2 - e^{2i\gamma} \sqrt{\xi} + e^{2i\alpha} s_{12}^2) \}.|.$$

To see the order of magnitude of the various terms we consider the case when Majorana phases vanish and the Dirac phases assume the value π . Then we get for vanishing $m_{e\mu}$,

$$s_{23} \lambda \chi_{13} - \lambda^2 (1 - \sqrt{\xi}) \chi_{14} \chi_{24} = 0. \quad (19)$$

In panel (a) of Fig. 5 we display the plot of $|m_{e\mu}|$ with m_3 for the 3 generation scenario i.e for $\sin^2 \theta_{24} = 0$ for IH. In panel (b) we consider the 3+1 case with all the parameters varying in their allowed range. Note that in the small m_3 limit (cf. Eq. 3.13) for $\alpha = 0$ the leading order term vanishes. For this case, for large active sterile mixing angle θ_{24} , the λ^2 term becomes large $\mathcal{O}(10^{-1})$ and the cancellation with λ term is not be possible. When CP violating phase α is non zero, the leading order term can cancel the λ^2 term even for large values of s_{24}^2 . These features are reflected in panel (c) where we plot $|m_{e\mu}|$ for $s_{24}^2 = 0.04$ and $\alpha = 0$ (blue/dark region) and by varying α in its full range (cyan/light region). As expected, for $\alpha = 0$, cancellation is not achieved for smaller values of m_3 . Thus the condition $|m_{e\mu}| = 0$ implies some correlation between m_3 and α for IH. Even if α is varied in its full range, the absolute value of the matrix element $|m_{e\mu}|$ can vanish only if the product $\chi_{14} \chi_{24}$ is small, i.e. s_{14}^2 and s_{24}^2 are simultaneously small. This is because if they are large the λ^2 term becomes of the $\mathcal{O}(10^{-1})$ and hence cancellation will not be possible. This is seen from panel (d) where for $s_{14}^2 = 0.06$ and $s_{24}^2 = 0.04$ the region where m_3 is small gets disallowed. Taking

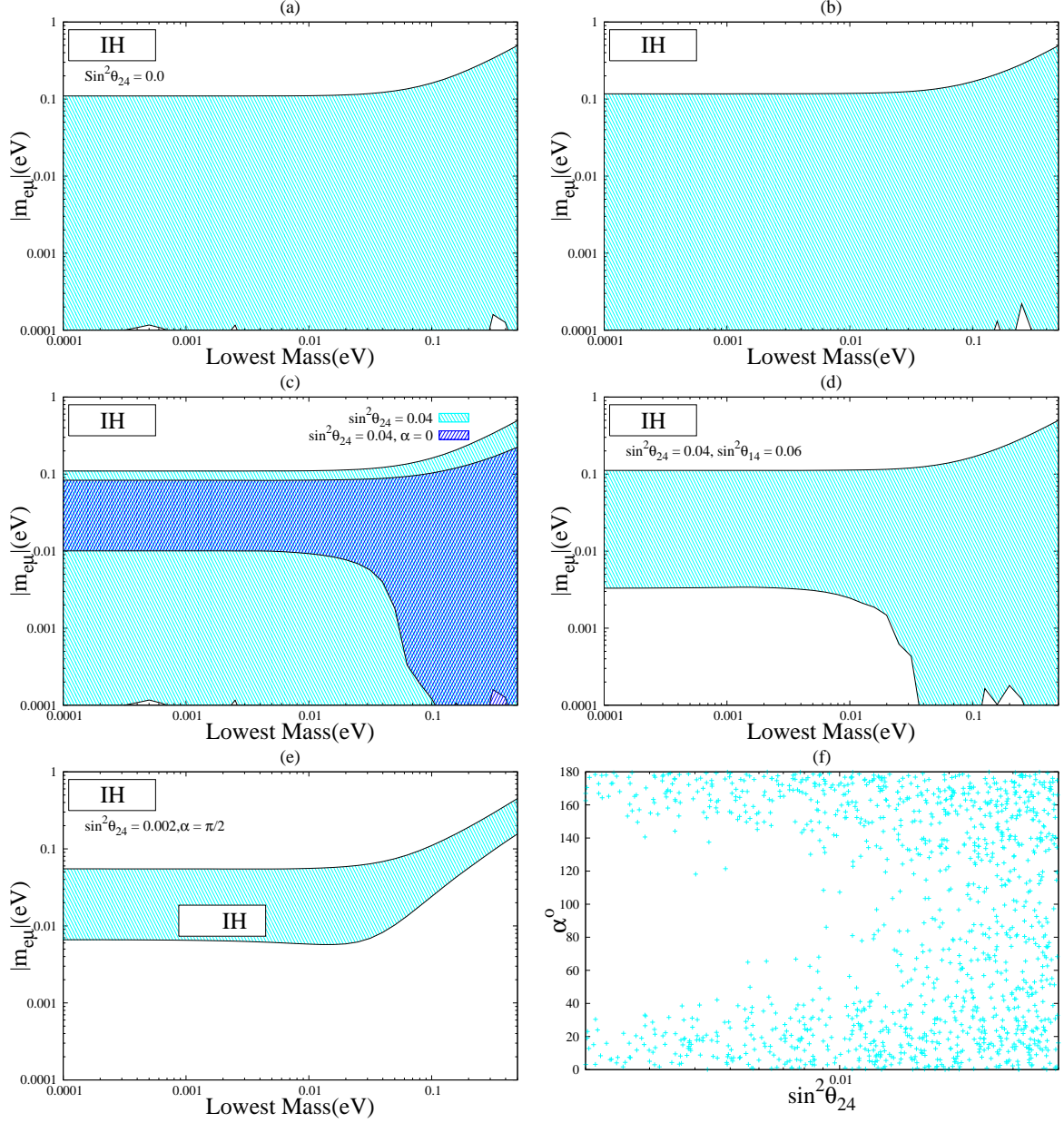


Figure 5: Plots of $|m_{e\mu}|$ vs m_3 for inverted hierarchy for (a) three generation case (b) 3+1 case with all parameters varied randomly in their full range. Panel (c), (d) and (e) are for specific values of s_{24}^2 and α . The panel (f) shows the correlation between α and s_{24}^2 when all other parameters are randomly varied.

CP violating phase $\alpha = \pi/2$ makes the magnitude of leading order term ($s_{12}c_{12}c_{23}\sqrt{\zeta}$) quite large and smaller values of θ_{24} cannot give cancellation even for large values of m_3 which can be seen from panel (e) of Fig. 5. For the occurrence of cancellation s_{24}^2 has to be ≥ 0.01 for $\alpha = \pi/2$ as can be seen from panel (f) where we have plotted the correlation between α

and s_{24}^2 for $|m_{e\mu}| = 0$.

C. The Mass Matrix element $m_{e\tau}$

The mass matrix element $m_{e\tau}$ in the presence of an extra sterile neutrino is

$$\begin{aligned}
m_{e\tau} = & c_{14}c_{24}e^{i(2\gamma+\delta_{14})}m_4s_{14}s_{34} + m_3c_{14}s_{13}e^{i(2\beta+\delta_{13})}(-c_{24}s_{13}s_{14}s_{34}e^{i(\delta_{14}-\delta_{13})} \\
& + c_{13}(c_{23}c_{34} - e^{i\delta_{24}}s_{23}s_{24}s_{34})) + m_2s_{12}c_{13}c_{14}e^{2i\alpha}(c_{12}(-c_{34}s_{23} - c_{23}s_{24}s_{34}e^{i\delta_{24}}) \\
& + s_{12}(-c_{13}c_{24}s_{14}s_{34}e^{i\delta_{14}} - e^{i\delta_{13}}s_{13}(c_{23}c_{34} - e^{i\delta_{24}}s_{23}s_{24}s_{34}))) \\
& + m_1c_{12}c_{13}c_{14}(-s_{12}(-c_{34}s_{23} - c_{23}s_{24}s_{34}e^{i\delta_{24}}) + c_{12}(-c_{13}c_{24}s_{14}s_{34}e^{i\delta_{14}} - e^{i\delta_{13}}s_{13} \\
& (c_{23}c_{34} - e^{i\delta_{24}}s_{23}s_{24}s_{34}))).
\end{aligned} \tag{20}$$

This element $m_{e\tau}$ and $m_{e\mu}$ are related by $\mu - \tau$ permutation symmetry

$$P_{\mu\tau} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

in such a way that

$$m_{e\tau} = P_{\mu\tau}^T m_{e\mu} P_{\mu\tau}.$$

For three active neutrino case the mixing angle θ_{23} in the partner textures linked by $\mu - \tau$ symmetry are related as $\bar{\theta}_{23} = (\frac{\pi}{2} - \theta_{23})$. However, in the 3+1 case the relation of θ_{23} between two textures related by this symmetry is not simple. The active sterile mixing angles θ_{24} and θ_{34} are also different in the textures connected by $\mu - \tau$ symmetry and are related as [17]

$$\bar{\theta}_{12} = \theta_{12}, \quad \bar{\theta}_{13} = \theta_{13}, \quad \bar{\theta}_{14} = \theta_{14}, \tag{21}$$

$$\sin \bar{\theta}_{24} = \sin \theta_{34} \cos \theta_{24}, \tag{22}$$

$$\sin \bar{\theta}_{23} = \frac{\cos \theta_{23} \cos \theta_{34} - \sin \theta_{23} \sin \theta_{34} \sin \theta_{24}}{\sqrt{1 - \cos \theta_{24}^2 \sin \theta_{34}^2}}, \tag{23}$$

$$\sin \bar{\theta}_{34} = \frac{\sin \theta_{24}}{\sqrt{1 - \cos \theta_{24}^2 \sin \theta_{34}^2}}. \tag{24}$$

Due to these relations the behaviour of $m_{e\mu}$ is different from that of $m_{e\tau}$ unlike in three active neutrino case where the plots of these two elements were same except for θ_{23} which

differed in octant for both the textures.

It is found that in the limit of small θ_{24} the two active sterile mixing angles $\bar{\theta}_{24} \approx \theta_{34}$ from Eq (22). The same can be seen from Eq (24) which gives $\bar{\theta}_{34} \approx \theta_{24}$ for smaller values of the mixing angle θ_{34} . Thus, for these cases the behaviour shown by θ_{24} in $m_{e\mu}$ ($m_{\mu\mu}$) is same as shown by θ_{34} in $m_{e\tau}$ ($m_{\tau\tau}$).

In the limit of vanishing active sterile mixing angle θ_{34} this element becomes

$$m_{e\tau} = c_{14}(m_{e\tau})_{3\nu}.$$

Using the approximation in Eq (2) for NH the above element can be expressed as,

$$\begin{aligned} |m_{e\tau}| \approx & \left| \sqrt{\Delta m_{23}^2} \{ -s_{12}s_{23}c_{12}c_{34}\sqrt{\zeta}e^{2i\alpha} + \lambda(c_{23}c_{34}e^{i(2\beta+\delta_{13})}\chi_{13} - c_{23}c_{34}s_{12}^2 \right. \\ & \chi_{13}\sqrt{\zeta}e^{i(2\alpha+\delta_{13})} + e^{i(2\gamma+\delta_{14})}\sqrt{\xi}s_{34}\chi_{14} - e^{i(2\alpha+\delta_{14})}\sqrt{\zeta}s_{12}^2s_{34}\chi_{14} - c_{12}c_{23} \\ & \left. e^{i(2\alpha+\delta_{24})}s_{12}s_{34}\chi_{24}\sqrt{\zeta} - e^{i(\delta_{13}+\delta_{24})}(e^{2i\beta} - e^{2i\alpha}s_{12}^2\sqrt{\zeta})s_{23}s_{34}\chi_{13}\chi_{24}\lambda^2 \} \right|. \end{aligned} \quad (25)$$

For a case of vanishing Majorana phases and Dirac phases having the value π this element can vanish when

$$\begin{aligned} - c_{12}c_{34}\sqrt{\zeta}s_{12}s_{23} - (1 - \sqrt{\zeta}s_{12}^2)s_{23}s_{34}\lambda^2\chi_{13}\chi_{24} + \lambda(-c_{23}c_{34}\chi_{13} + \\ \sqrt{\zeta}c_{23}s_{12}(c_{34}s_{12}\chi_{13} + c_{12}s_{34}\chi_{24}) + s_{12}^2s_{34}\chi_{14}\sqrt{\zeta} - \sqrt{\xi}s_{34}\chi_{14}) = 0. \end{aligned} \quad (26)$$

For a vanishing active sterile mixing angle θ_{34} one recovers the 3 generation case. In this limit, from Eq. (26) one observes that the leading order term and the term with λ are of the same order $\sim \mathcal{O}(10^{-2})$ while the λ^2 term vanishes and hence cancellation is possible excepting for very low values of the lightest mass. We can see this in panel (a) of Fig. 6. In panel (b) (red/light region) all the parameters are varied randomly (3+1 case) and cancellation is seen to be possible over the whole range of m_1 . In panel (b) (green/dark region) we also plot the element $|m_{e\tau}|$ for the upper limit of $s_{34}^2 = 0.18$. In this case there is no cancellation for very low values of the smallest mass. This is because when s_{34}^2 is large, the λ term containing ξ becomes large $\mathcal{O}(1)$ and there will be no cancellation.

For inverted hierarchy the element $m_{e\tau}$ using the approximation in Eq (3) becomes

$$\begin{aligned} |m_{e\tau}| \approx & \left| \sqrt{\Delta m_{23}^2} \{ c_{12}c_{34}s_{12}s_{23}(-e^{2i\alpha} + 1) + e^{i(\delta_{13}+\delta_{24})}(c_{12}^2 + e^{2i\alpha}s_{12}^2)s_{23}s_{34}\lambda^2\chi_{13}\chi_{24} \right. \\ & - \lambda(c_{23}c_{34}\chi_{13}e^{i\delta_{13}}(c_{12}^2 + e^{2i\alpha}s_{12}^2) + e^{i\delta_{14}}s_{34}\chi_{14}(c_{12}^2 + e^{i\alpha}s_{12}^2) \\ & \left. - e^{i(2\gamma+\delta_{14})}s_{34}\chi_{14}\sqrt{\xi} + c_{12}c_{23}s_{12}s_{34}\chi_{24}e^{i\delta_{24}}(e^{2i\alpha} - 1)) \} \right| \end{aligned} \quad (27)$$

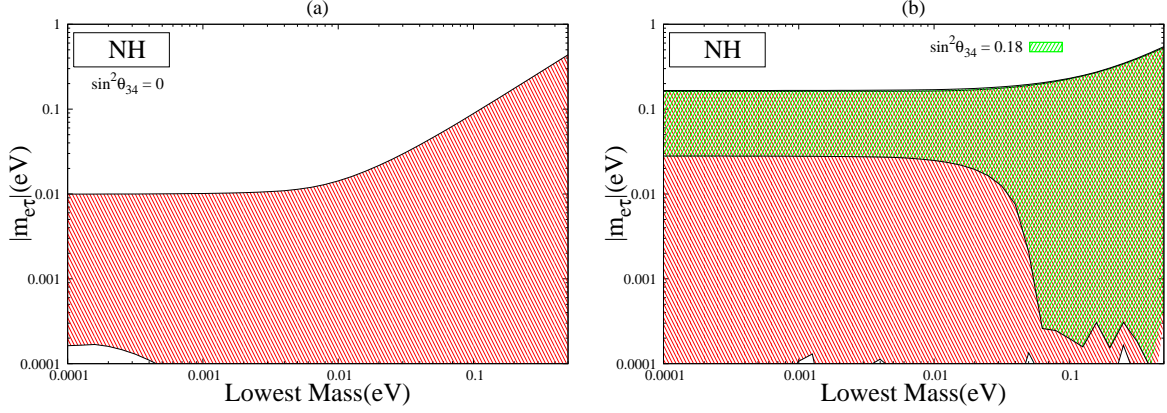


Figure 6: Plots of $|m_{e\tau}|$ for normal hierarchy with lowest mass m_1 . The panel (a) corresponds to three generation case. In (b) (red/light region) all the parameters are varied in their full allowed range and the green/dark region is for $s_{34}^2 = 0.18$ with all the other parameters covering their full range.

In the limit of vanishing Majorana phases and Dirac CP violating phases equal to π this element becomes negligible when

$$\lambda(c_{23}c_{34}\chi_{13} + s_{34}\chi_{14} - s_{34}\chi_{14}\sqrt{\xi}) + s_{23}s_{34}\chi_{13}\chi_{24}\lambda^2 = 0. \quad (28)$$

In panel (a) of Fig. 7 the three generation case is reproduced by putting $s_{34}^2 = 0$ and in (b) all the parameters are varied in their allowed range (3+1 case). In both the figures we can see that cancellation is permissible over the whole range of m_3 considered. When the CP violating phase $\alpha = 0$ we see that the leading order term ($\sin 2\theta_{12}s_{23}c_{34}$) vanishes and as a result for large values of s_{34}^2 the cancellation is not possible because the term with coefficient λ becomes large ($\mathcal{O}(10^{-1})$). For non zero values of the CP violating phase α this leading order term is non zero and its contribution will be significant. So in this case high values of θ_{34} are also allowed because now the leading order and the term with coefficient λ will be of same magnitude. When we fix $s_{34}^2 = 0.06$ and $\alpha = 0$ the region where m_3 is small is disallowed (panel (c) blue/dark region) but when α varies within its full range the disallowed regions become allowed (panel (c) cyan/light region). When s_{34}^2 approaches its upper limit the λ term having ξ becomes very large and cancellation is not possible even for non zero values of α which can be seen from panel (d). However, when $\alpha = \pi/2$ very small values of s_{34}^2 cannot give cancellation as the leading order term becomes large (panel (e)). s_{34}^2 has to be ≥ 0.01 for the term to vanish which can be seen from panel (f) where we plotted the correlation between α and s_{34}^2 for $|m_{e\tau}| = 0$.

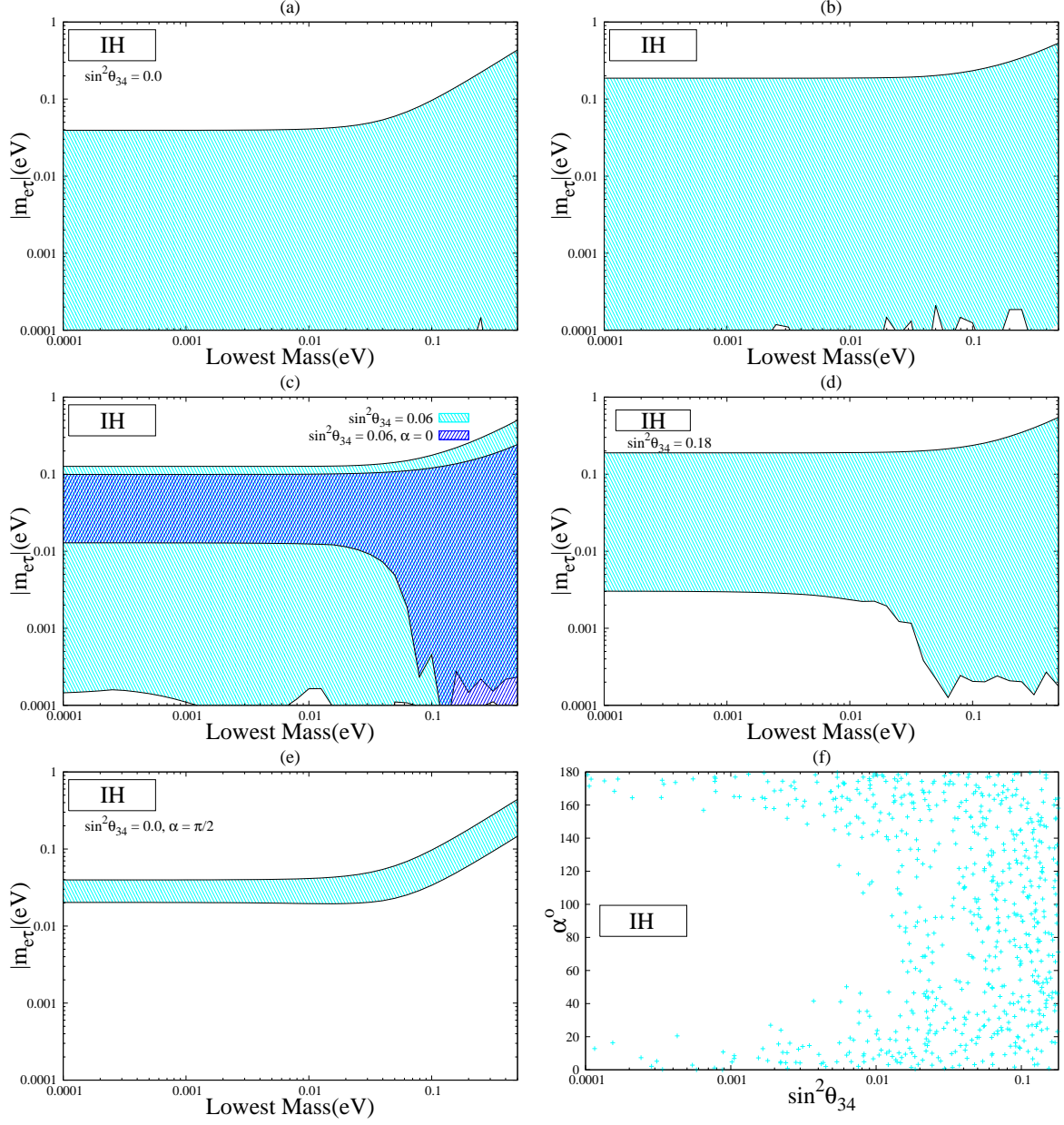


Figure 7: Plots of $|m_{e\tau}|$ for inverted hierarchy with lowest mass m_3 . The panel (a) corresponds to three generation case. In (b) all the parameters are varied in their full allowed range (3+1). The panel (c), (d) is for specific value of θ_{34} and α with all the other parameters covering their full range. The panel (f) shows correlation between α and s_{24}^2 .

D. The Mass Matrix element $m_{\mu\mu}$

This diagonal neutrino mass matrix is given as

$$\begin{aligned}
m_{\mu\mu} = & e^{2i(\delta_{14}-\delta_{24}+\gamma)} c_{14}^2 m_4 s_{24}^2 \\
& + e^{2i(\delta_{13}+\beta)} m_3 (c_{13} c_{24} s_{23} - e^{i(\delta_{14}-\delta_{13}-\delta_{24})} s_{13} s_{14} s_{24})^2 \\
& + m_1 \{-c_{23} c_{24} s_{12} + c_{12} (-e^{i\delta_{13}} c_{24} s_{13} s_{23} - e^{i(\delta_{14}-\delta_{24})} c_{13} s_{14} s_{24})\}^2 \\
& + e^{2i\alpha} m_2 \{c_{12} c_{23} c_{24} + s_{12} (-e^{i\delta_{13}} c_{24} s_{13} s_{23} - e^{i(\delta_{14}-\delta_{24})} c_{13} s_{14} s_{24})\}^2
\end{aligned} \tag{29}$$

This expression reduces to its three generation case if the mixing angle θ_{24} vanishes. Also we can see from the expression that there is no dependence on the mixing angle θ_{34} . Using the approximation in Eqs. (2) this element can be simplified to the form

$$\begin{aligned}
|m_{\mu\mu}| \approx & |\sqrt{\Delta m_{23}^2} \{c_{12}^2 c_{23}^2 e^{2i\alpha} \sqrt{\zeta} + e^{i(\delta_{13}+2\beta)} s_{23}^2 \\
& - 2\lambda c_{12} c_{23} e^{i(\delta_{13}+2\alpha)} \sqrt{\zeta} s_{12} s_{23} \chi_{13} \\
& + \lambda^2 \{e^{2i(\delta_{13}+\alpha)} \sqrt{\zeta} s_{12}^2 s_{23}^2 \chi_{13}^2 + e^{i(\delta_{14}-\delta_{24})} (e^{i(2\gamma+\delta_{14}-\delta_{24})} \sqrt{\xi} \chi_{24} \\
& - 2e^{2i\alpha} \sqrt{\zeta} c_{12} c_{23} s_{12} \chi_{14}) \chi_{24}\} \}|.
\end{aligned} \tag{30}$$

For a case of vanishing Majorana CP phases and the Dirac phases having the value π this element vanishes when

$$\begin{aligned}
& s_{23}^2 + c_{12}^2 c_{23}^2 \sqrt{\zeta} + c_{12} s_{12} \sin 2\theta_{23} \sqrt{\zeta} \lambda \chi_{13} \\
& + \lambda^2 (s_{12}^2 s_{23}^2 \sqrt{\zeta} \chi_{13}^2 - c_{23} \sin 2\theta_{12} \sqrt{\zeta} \chi_{14} \chi_{24} + \sqrt{\xi} \chi_{24}^2) = 0.
\end{aligned} \tag{31}$$

We know that for the case of 3 generation, the elements in the $\mu - \tau$ block are quite large and cannot vanish for normal hierarchy. In panel (a) of Fig. 8 we can see that $|m_{\mu\mu}|$ cannot vanish in small m_1 region for $s_{24}^2 = 0$ which is indeed the 3 generation case. This is because the magnitude of the first two terms in Eq. (31) is quite large in this case, $\sim \mathcal{O}(10^{-1})$ and for cancellation to occur the term with coefficient λ^2 has to be of the same order and this is not possible when s_{24}^2 is small. However when s_{24}^2 varied in its full allowed range the contribution of the sterile part is enhanced and this can cancel the active part as can be seen from panel (b). Now to understand the dependence of $m_{\mu\mu}$ with θ_{24} , if we increase s_{24}^2 from its lower bound we can see that the two terms become of the same order. So there will be region in the limit of small m_1 for which this element vanishes (panel (c)). We see in panel (d) of Fig. 8 that when θ_{24} acquires very large values, than the magnitude of the $\lambda^2 (\sqrt{\xi} \chi_{24}^2)$ term becomes large thus, leading to non cancellation of the terms with the leading order first two terms. Hence, the region with very small m_1 is not allowed. Using the approximation

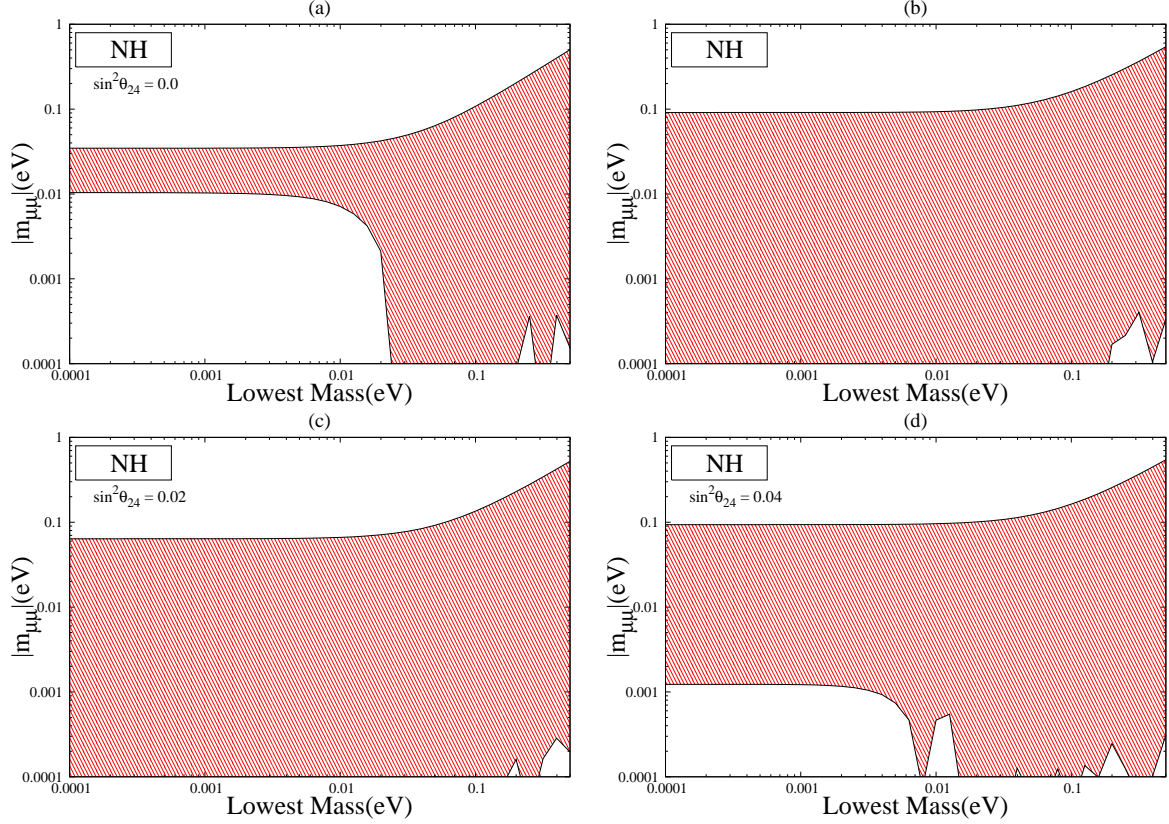


Figure 8: Plots of vanishing $m_{\mu\mu}$ for normal hierarchy for different values of θ_{24} when all other mixing angles are varied in their 3σ ranges and CP violating phases are varied in their full range.

for inverted hierarchy the element $m_{\mu\mu}$ becomes

$$\begin{aligned}
 |m_{\mu\mu}| \approx & \left| \sqrt{\Delta m_{23}^2} \{ c_{23}^2 (s_{12}^2 + c_{12}^2 e^{2i\alpha}) + \frac{1}{2} \lambda \sin 2\theta_{12} \sin 2\theta_{23} e^{i\delta_{13}} (1 - e^{2i\alpha}) \chi_{13} \right. \\
 & + \lambda^2 [\sin 2\theta_{12} c_{23} e^{i(\delta_{14} - \delta_{24})} (1 - e^{2i\alpha}) \chi_{14} \chi_{24} + s_{23}^2 e^{2i\delta_{13}} (c_{12}^2 + e^{2i\alpha} s_{12}^2) \chi_{13}^2 \\
 & \left. + e^{2i(\gamma + \delta_{14} - \delta_{24})} \sqrt{\xi} \chi_{24}^2] \right|.
 \end{aligned} \quad (32)$$

For a case of vanishing Majorana phases and Dirac phases having value π , this element can vanish when

$$c_{23}^2 + \lambda^2 (s_{23}^2 \chi_{13}^2 + \sqrt{\xi} \chi_{24}^2) = 0. \quad (33)$$

In panel (a) of Fig. 9 we plotted $|m_{\mu\mu}|$ for $s_{24}^2 = 0$ to reproduce the case 3 generation whereas in panel (b) all the parameters are varied in their allowed range in 3+1 scenario. In both the case we can see that cancellation is possible for full range of m_3 . It can be noticed that unlike normal hierarchy here cancellation is possible for small values of s_{24}^2 it because in this case all the terms are of same order and there can always be cancellation. However, if we

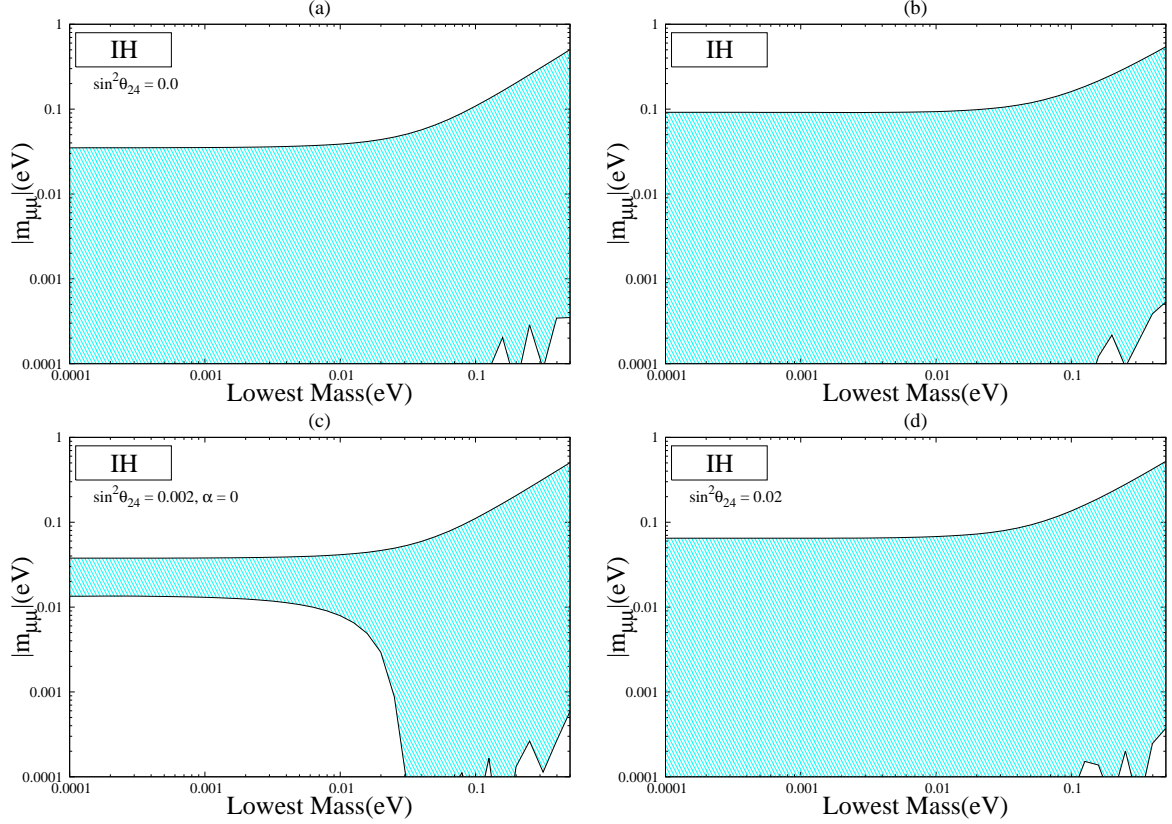


Figure 9: Plots of vanishing $m_{\mu\mu}$ for inverted hierarchy with lowest mass m_3 . Panel (a) for 3 generation case (b) all the parameters are varied in full allowed range (3+1). Panel (c) and (d) are for specific values of α and s_{24}^2 are taken with all other parameters covering their full range.

put $\alpha = 0$ then the term $\lambda (\sin 2\theta_{12}s_{23}c_{23}\chi_{13})$ drops out from the equation and the leading order term can not be canceled for small values of s_{24}^2 . It can be seen from panel (c) that for $s_{24}^2 = 0.002$ and $\alpha = 0$ the regions where m_3 is small is not allowed. As the value of θ_{24} increases there is the possibility of cancellation of terms for all the values of α as can be seen from panel (d) where we plot $|m_{\mu\mu}|$ with the lowest mass for $s_{24}^2 = 0.02$ when all the other mixing angles are varied in 3σ range and CP violating phases are varied in full range. Now if we keep increasing s_{24}^2 then λ^2 term will become large and the chance of cancellation will be less.

E. The Mass Matrix element $m_{\mu\tau}$

The (2,3) element of M_ν in the flavor basis becomes quite complicated in the presence of an extra sterile neutrino. The expression is

$$\begin{aligned}
m_{\mu\tau} = & e^{i(2\delta_{14}-\delta_{24}+2\gamma)} c_{14}^2 c_{24} m_4 s_{24} s_{34} \\
& + e^{2i(\delta_{13}+\beta)} m_3 (c_{13} c_{24} s_{23} - e^{i(\delta_{14}-\delta_{24}-\delta_{13})} s_{13} s_{14} s_{24}) \\
& \quad \{ -e^{i(\delta_{14}-\delta_{13})} c_{24} s_{13} s_{14} s_{34} + c_{13} (c_{23} c_{34} - e^{i\delta_{24}} s_{23} s_{24} s_{34}) \} \\
& + m_1 \{ -c_{23} c_{24} s_{12} + c_{12} (-e^{i\delta_{13}} c_{24} s_{13} s_{23} - e^{i(\delta_{14}-\delta_{24})} c_{13} s_{14} s_{24}) \} \\
& \quad [-s_{12} (-c_{34} s_{23} - e^{i\delta_{24}} c_{23} s_{24} s_{34}) \\
& + c_{12} \{ -e^{i\delta_{14}} c_{13} c_{24} s_{14} s_{34} - e^{i\delta_{13}} s_{13} (c_{23} c_{34} - e^{i\delta_{24}} s_{23} s_{24} s_{34}) \}] \\
& + e^{2i\alpha} m_2 \{ c_{12} c_{23} c_{24} + s_{12} (-e^{i\delta_{13}} c_{24} s_{13} s_{23} - e^{i(\delta_{14}-\delta_{24})} c_{13} s_{14} s_{24}) \} \\
& \quad [c_{12} (-c_{34} s_{23} - e^{i\delta_{24}} c_{23} s_{24} s_{34}) \\
& + s_{12} \{ -e^{i\delta_{14}} c_{13} c_{24} s_{14} s_{34} - e^{i\delta_{13}} s_{13} (c_{23} c_{34} - e^{i\delta_{24}} s_{23} s_{24} s_{34}) \}].
\end{aligned} \tag{34}$$

It reduces to the 3 generation case when $\theta_{24} = \theta_{34} = 0$. In the normal hierarchical region where m_1 can assume very small values and can be neglected, using approximations in Eqs. (2, 13, 14) we get

$$\begin{aligned}
|m_{\mu\tau}| \approx & |\sqrt{\Delta m_{23}^2} \{ c_{23} c_{34} (e^{2i(\beta+\delta_{13})} - e^{2i\alpha} \sqrt{\zeta} c_{12}^2) s_{23} \\
& - \lambda [c_{12} c_{34} e^{i(2\alpha+\delta_{13})} \sqrt{\zeta} s_{12} \cos 2\theta_{23} \chi_{13} + e^{i\delta_{24}} (e^{2i\alpha} c_{12}^2 c_{23}^2 \sqrt{\zeta} + e^{2i(\beta+\delta_{13})} s_{23}^2) \chi_{24} s_{34} \\
& - e^{2i\delta_{14}} (e^{i(2\gamma-\delta_{24})} \sqrt{\xi} \chi_{24} - c_{12} c_{23} e^{2i\alpha} \sqrt{\zeta} s_{12} \chi_{14}) s_{34}] \\
& + \lambda^2 [\sqrt{\zeta} e^{i(2\alpha+\delta_{13})} (e^{i\delta_{14}} s_{12} \chi_{14} + 2c_{12} c_{23} e^{i\delta_{24}} \chi_{24}) s_{12} s_{23} s_{34} \chi_{13} \\
& + e^{i\delta_{14}} (e^{i(2\alpha-\delta_{24})} c_{12} c_{34} \sqrt{\zeta} s_{12} \chi_{24} - e^{i(2\beta+\delta_{13})} \chi_{13} s_{34}) \chi_{14} s_{23} \\
& + c_{23} c_{34} e^{2i(\alpha+\delta_{13})} s_{12}^2 \chi_{13}^2 \sqrt{\zeta} s_{23} \}]|.
\end{aligned} \tag{35}$$

To see the order of terms we consider the case where Majorana CP phases vanish and Dirac phases have the value π this element becomes negligible when

$$\begin{aligned}
& c_{23} c_{34} s_{23} (1 - c_{12}^2 \sqrt{\zeta}) + \lambda \{ (c_{12} c_{34} s_{12} \sqrt{\zeta} \chi_{13}) \cos 2\theta_{23} \\
& + \chi_{24} s_{34} (s_{23}^2 + c_{12}^2 c_{23}^2 \sqrt{\zeta}) + s_{34} (\sqrt{\xi} \chi_{24} + c_{12} c_{23} s_{12} \sqrt{\zeta} \chi_{14}) \} \\
& + \lambda^2 \{ s_{12} \chi_{13} s_{23} s_{34} \sqrt{\zeta} (s_{12} \chi_{14} + 2c_{12} c_{23} \chi_{24}) + \chi_{14} s_{23} (c_{12} c_{34} s_{12}^2 s_{23} \sqrt{\zeta} \chi_{13}^2) \} = 0.
\end{aligned} \tag{36}$$

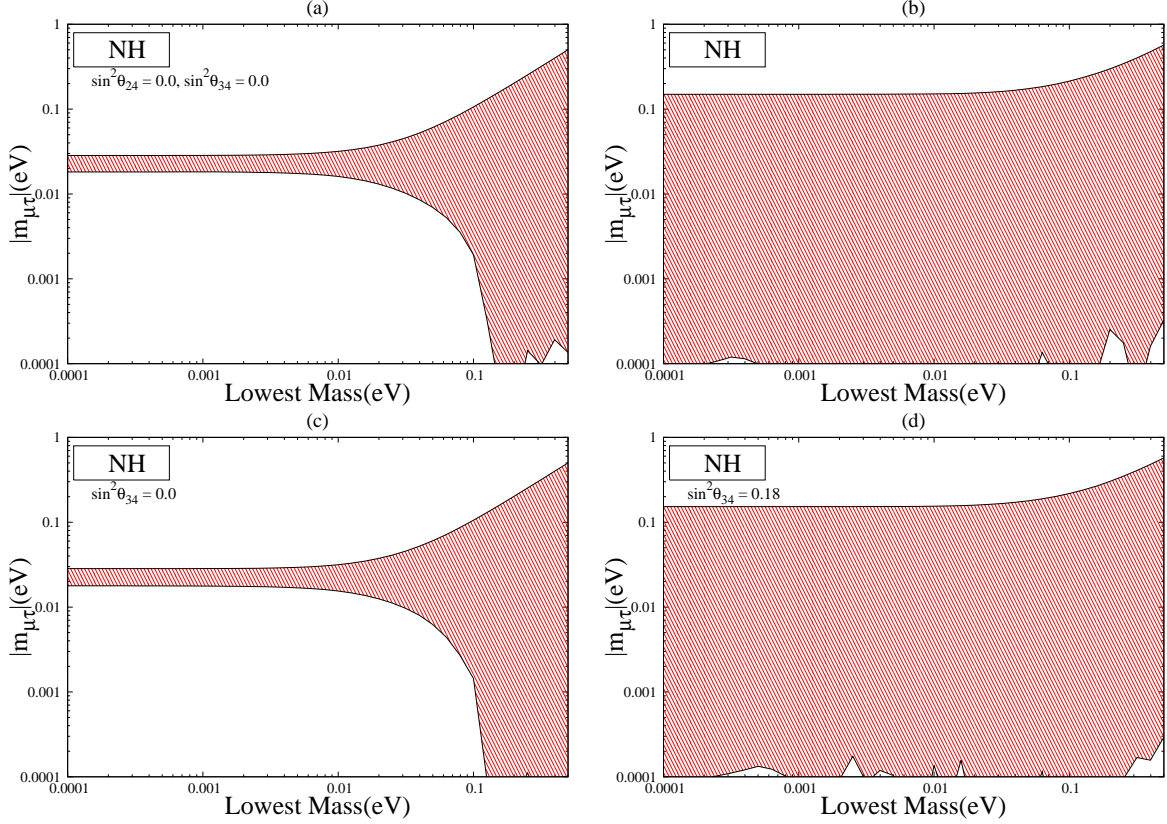


Figure 10: Plots of vanishing $m_{\mu\tau}$ for normal hierarchy (a) for vanishing θ_{34} and θ_{24} . In panel (b) all parameters are varied in their full allowed range. Panel(c, d) are for specific values of θ_{34} when all other mixing angles are varied in their full range.

Being an element of $\mu\tau$ block, $m_{\mu\tau}$ shows the same behaviour that of $m_{\mu\mu}$ in normal hierarchy. In panel (a) of Fig. 10 we plotted $|m_{\mu\tau}|$ for $s_{24}^2 = s_{34}^2 = 0$ to coincide with the 3 generation case and we can see that cancellation is not possible in hierarchical region. However, when all the parameters are varied in their allowed range in panel (b) it get contribution from the sterile part and cancellation is always possible. It can also be seen from panel (c) of Fig. 10 that $s_{34}^2 = 0$ is also unable to give cancellation in the region when m_1 is small and the figure is quite similar to that of 3 generation case. However, as this active sterile mixing angle becomes larger there is always a possibility of allowed region towards the lower values of m_1 as is evident from panel (d). This is because for the vanishing value of θ_{34} the terms with λ and λ^2 become very small and cannot cancel the leading term $\mathcal{O}(10^{-1})$. It can also be seen that in this case (i.e $s_{34}^2 = 0$), there is no χ_{24} term in Eq. (3.31) and this is why the figure is somewhat similar to the 3 generation case. However, when θ_{34} increases these two contributions become large and thus cancellation becomes possible. For the case of inverted

hierarchy where m_3 can have very small values, $m_{\mu\tau}$ becomes

$$\begin{aligned}
|m_{\mu\tau}| \approx & \left| \sqrt{\Delta m_{23}^2} \{ -c_{23}c_{34}s_{23}(c_{12}^2 e^{2i\alpha} + s_{12}^2) \right. \\
& + \lambda [c_{12}s_{12}(1 - e^{2i\alpha})(c_{34} \cos 2\theta_{23} e^{i\delta_{13}} \chi_{13} + c_{23}s_{34} e^{i\delta_{14}} \chi_{14}) \\
& + s_{34} \{ e^{i(2\gamma+2\delta_{14}-\delta_{24})} \sqrt{\xi} - c_{23}^2 e^{i\delta_{24}} (s_{12}^2 + c_{12}^2 e^{2i\alpha}) \} \chi_{24}] \\
& + \lambda^2 [c_{23}c_{34}s_{23} e^{2i\delta_{13}} (c_{12}^2 + e^{2i\alpha} s_{12}^2) \chi_{13}^2 \\
& \left. + c_{12}s_{12}s_{23}(e^{2i\alpha} - 1)(c_{34} e^{i(\delta_{14}-\delta_{24})} \chi_{14} + 2s_{34}c_{23} e^{i(\delta_{13}+\delta_{24})} \chi_{13}) \chi_{24} \} \right|.
\end{aligned} \tag{37}$$

To get an idea about the magnitude of the terms we take the vanishing Majorana phases and Dirac CP phases to be of the order π . The expression in this case for vanishing $m_{\mu\tau}$ becomes

$$-c_{23}c_{34}s_{23} + \lambda(s_{34}\chi_{24}(c_{23}^2 - \sqrt{\xi})) - \lambda^2(-s_{34}\chi_{13}\chi_{14} - c_{23}c_{34}\chi_{13}^2) = 0 \tag{38}$$

In panel (a) of Fig. 11 where $|m_{\mu\tau}|$ for 3 generation is plotted we can see that unlike $m_{\mu\mu}$ there is no cancellation in small m_3 region but when plotted for the full range it gets contribution from the sterile part and there is cancellation for the full range of m_3 (panel (b)). Clearly the cancellation of the terms do not become possible for small values of θ_{34} in strict hierarchical region. This case is similar to the three generation case in IH (cyan/light region, panel (c)). This is because for $s_{34}^2 = 0$ the contribution of s_{24}^2 comes from the λ^2 term. If we put the CP violating phase α as zero then cancellation is not possible for whole range of m_3 (blue/dark region panel (c)). However, as the value of s_{34}^2 increases all the terms in the above equation becomes of the same order and cancellation for very small values of m_3 is possible (panel (d)).

F. The Mass Matrix element $m_{\tau\tau}$

This element is related to $m_{\mu\mu}$ by the $\mu - \tau$ symmetry. As discussed earlier, in the limit when θ_{24} and θ_{34} are not very large, the two mixing angles θ_{34} and θ_{24} will behave same in the textures related by $\mu - \tau$ symmetry. The (3,3) element of the neutrino mass matrix in the presence of one sterile neutrino is given as

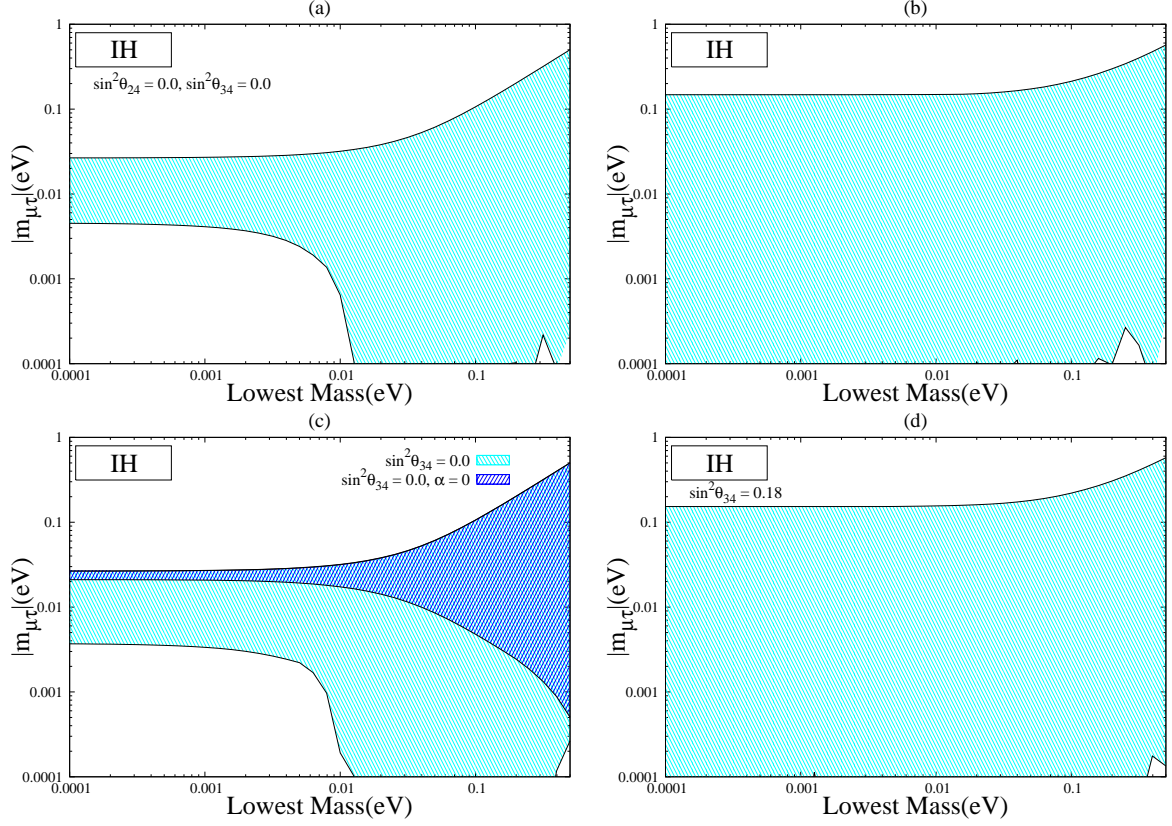


Figure 11: Plots of vanishing $m_{\mu\tau}$ for inverted hierarchy (a) for vanishing θ_{34} and θ_{24} (3 generation). In panel (b) all parameters are varied in their full allowed range (3+1). Panel (c) and (d) are for specific values of θ_{34} and α when all other mixing angles are varied in their full range.

$$\begin{aligned}
m_{\tau\tau} = & e^{2i(\delta_{14}+\gamma)} c_{14}^2 c_{24}^2 m_4 s_{34}^2 \\
& + e^{2i(\delta_{13}+\beta)} m_3 \{ e^{i(\delta_{14}-\delta_{13})} c_{24} s_{13} s_{14} s_{34} + c_{13} (c_{23} c_{34} - e^{i\delta_{24}} s_{23} s_{24} s_{34}) \}^2 \\
& + m_1 [-s_{12} (-c_{34} s_{23} - e^{i\delta_{24}} c_{23} s_{24} s_{34}) \\
& + c_{12} \{ -e^{i\delta_{14}} c_{13} c_{24} s_{14} s_{34} - e^{i\delta_{13}} s_{13} (c_{23} c_{34} - e^{i\delta_{24}} s_{23} s_{24} s_{34}) \}]^2 \\
& + e^{2i\alpha} m_2 [c_{12} (-c_{34} s_{23} - e^{i\delta_{24}} c_{23} s_{24} s_{34}) \\
& + s_{12} \{ -e^{i\delta_{14}} c_{13} c_{24} s_{14} s_{34} - e^{i\delta_{13}} s_{13} (c_{23} c_{34} - e^{i\delta_{24}} s_{23} s_{24} s_{34}) \}]^2.
\end{aligned} \tag{39}$$

It reduces to the 3 generation case for $\theta_{34} = 0$. Using the approximation for normal hierarchy in Eqs. (2, 13, 14) this becomes

$$\begin{aligned}
|m_{\tau\tau}| \approx & \left| \sqrt{\Delta m_{23}^2} \{ c_{23} c_{34} s_{23} (e^{2i\beta+\delta_{13}} - c_{12}^2 \sqrt{\zeta} e^{2i\alpha}) + \lambda \{ -e^{i(2\alpha+\delta_{13})} \sqrt{\zeta} \right. \\
& s_{12} c_{12} c_{34} \cos 2\theta_{23} \chi_{13} - \sqrt{\zeta} c_{12} c_{23} s_{34} e^{2i\alpha} (s_{12} \chi_{14} e^{i\delta_{14}} + c_{12} c_{23} \chi_{24} e^{2i\delta_{24}}) + s_{34} \chi_{24} (-s_{23}^2 e^{2i(\beta+\delta_{13})+i\delta_{24}} \\
& - \sqrt{\xi} e^{2i(\gamma+\delta_{14})-i\delta_{24}}) \} + \lambda^2 \{ \sqrt{\zeta} s_{12}^2 s_{23} \chi_{13} e^{i(2\alpha+\delta_{13})} (c_{23} c_{34} \chi_{13} e^{i\delta_{13}} + s_{34} \chi_{14} e^{i\delta_{14}}) \\
& + \sqrt{\zeta} c_{12} s_{12} s_{23} \chi_{24} e^{2i\alpha} (2c_{23} s_{34} \chi_{13} e^{i(\delta_{13}+\delta_{24})} + c_{34} \chi_{14} e^{i(\delta_{14}-\delta_{24})}) \\
& \left. - s_{23} s_{34} \chi_{13} \chi_{14} e^{i(2\beta+\delta_{13}+\delta_{14})} \} \right|.
\end{aligned} \tag{40}$$

To get an idea of the order of the terms we consider the vanishing Majorana phases and the Dirac phases having the value equal to π . This element vanishes when

$$\begin{aligned}
& c_{23} c_{34} s_{23} (1 - c_{12}^2 \sqrt{\zeta}) + \lambda \{ \sqrt{\zeta} s_{12} c_{12} c_{34} \cos 2\theta_{23} \chi_{13} + \sqrt{\zeta} c_{12} c_{23} s_{34} (s_{12} \chi_{14} \\
& + c_{12} c_{23} \chi_{24}) - s_{34} \chi_{24} (s_{23}^2 - \sqrt{\xi}) \} + \lambda^2 \{ \sqrt{\zeta} s_{12}^2 s_{23} \chi_{13} (c_{23} c_{34} \chi_{13} + s_{34} \chi_{14}) \\
& + \sqrt{\zeta} c_{12} s_{12} s_{23} \chi_{24} (2c_{23} s_{34} \chi_{13} + c_{34} \chi_{14}) + s_{23} s_{34} \chi_{13} \chi_{14} \} = 0.
\end{aligned} \tag{41}$$

It is found that for vanishing θ_{34} which is the case for 3 generation is disallowed for small m_1 as can be seen from panel (a) of Fig. 12. This is the generic behaviour of a element belonging to the $\mu - \tau$ block in normal hierarchy which we mentioned previously. This is because for θ_{34} equal to zero the leading order term is large ($\mathcal{O}(10^{-1})$). Here the term with λ^2 is quite small (10^{-3} - 10^{-4}) and hence will not have very significant role to play. Thus, only term with coefficient λ can cancel the leading order term. However, for vanishing θ_{34} this term is small $\mathcal{O}(10^{-3})$, thus not leading to any cancellation of the leading order term. In panel (b) when all the parameters are varied in their 3σ range we can see that cancellation is possible over the whole range of m_1 (3+1 case). Now, when θ_{34} starts increasing from its lowest value there exist a region for intermediate values where both the terms become approximately of the same order and hence there can be cancellations (panel (c)). Towards very large values of θ_{34} the term with coefficient λ becomes larger than the leading order term due to which this element cannot vanish. For the cancellation very large values of m_1 is required as can be seen from panel (d) of Fig 12. For the case of inverted hierarchy where m_3 approaches small values we get the expression

$$\begin{aligned}
m_{\tau\tau} \approx & c_{34}^2 s_{23}^2 (c_{12}^2 e^{2i\alpha} + s_{12}^2) + e^{2i(\delta_{14}+\gamma)} \sqrt{\xi} s_{34}^2 \\
& + 2\lambda [(e^{2i\alpha} - 1) c_{12} c_{34} s_{12} s_{23} (c_{23} c_{34} s_{12} s_{23} (c_{23} c_{34} e^{2i\delta_{13}} \chi_{13} + e^{2i\delta_{14}} s_{34} \chi_{14}) \\
& + 2c_{23} c_{34} e^{2i\delta_{24}} s_{23} s_{34} (c_{12}^2 e^{2i\alpha} + s_{12}^2) \chi_{24}] \\
& + \lambda^2 [(c_{12}^2 + e^{2i\alpha} s_{12}^2) \{ c_{23} c_{34} \chi_{13} e^{i\delta_{13}} (c_{23} c_{34} \chi_{13} e^{i\delta_{13}} + 2\chi_{14} s_{34} e^{i\delta_{14}}) + e^{2i\delta_{14}} \chi_{14}^2 s_{34}^2 \}
\end{aligned} \tag{42}$$

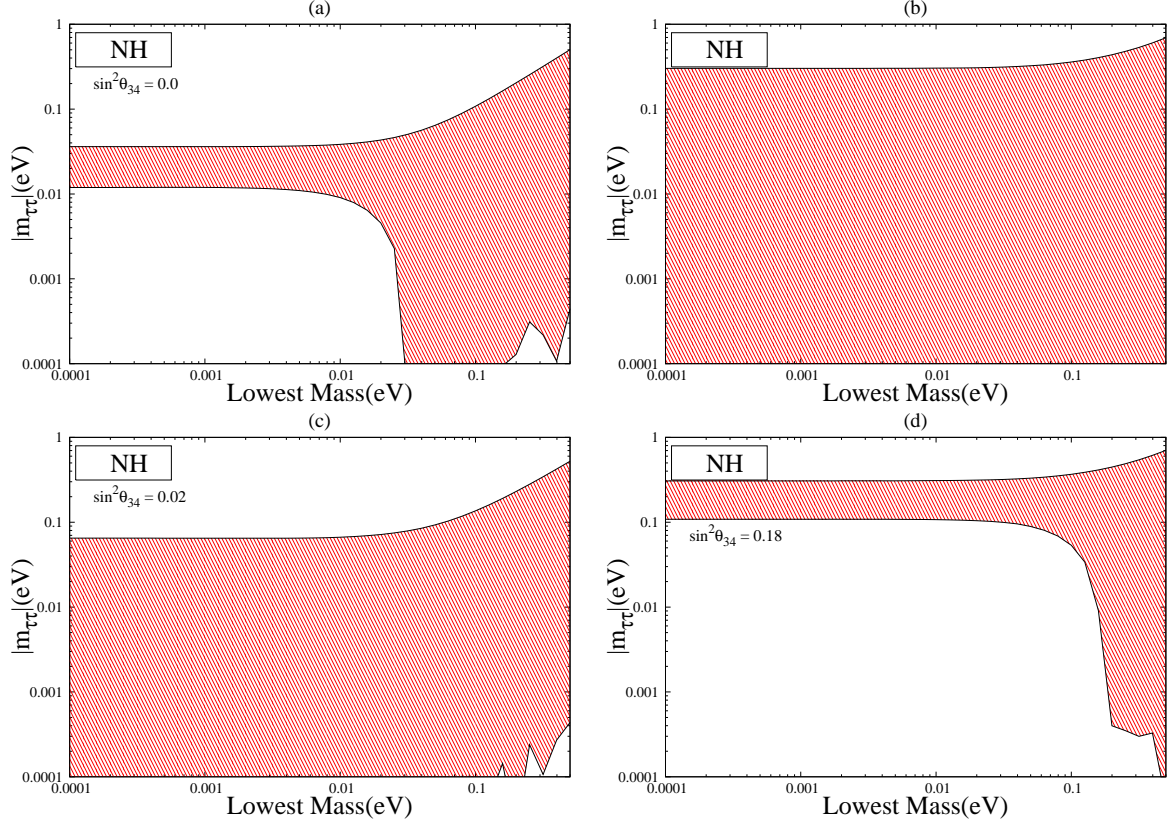


Figure 12: Plots of vanishing $m_{\tau\tau}$ for normal hierarchy with lowest mass m_1 . Panel (a) correspond to three generation case. In panel (b) all the mixing angles are varied in their full allowed range of parameters (3+1). Panel (c) and (d) are for some specific values of θ_{34} .

$$\begin{aligned}
& + (c_{12}^2 e^{2i\alpha} + s_{12}^2) c_{23}^2 e^{2i\delta_{24}} \chi_{24}^2 s_{34}^2 \\
& + 2s_{12}(e^{2i\alpha} - 1)e^{i\delta_{24}}(c_{34}\chi_{13} \cos 2\theta_{23} e^{i\delta_{13}} + c_{12}c_{23}\chi_{14}s_{34})s_{34}\chi_{24}].
\end{aligned}$$

For vanishing Majorana CP phases and Dirac phases having the value equal to π this expression becomes

$$m_{\tau\tau} \approx -c_{23}c_{34}s_{23} + \lambda s_{34}\chi_{24}(c_{23}^2 - \sqrt{\xi}) + \lambda^2 s_{23}\chi_{13}(c_{23}c_{34}\chi_{13} + s_{34}\chi_{14}) \quad (43)$$

In panel (a) of Fig. 13 we reproduced the 3 generation behaviour by plotting $|m_{\tau\tau}|$ for $s_{34}^2 = 0$ and in panel (b) all the parameters are varied randomly (3+1). In both the cases we can see that cancellations are possible for the whole range of m_3 . For $s_{34}^2 = 0$ all the terms are of same order and cancellations are always possible. But if we put $\alpha = 0$ then one term with coefficient λ and another term with coefficient λ^2 drops out from the equation and then small values of s_{34}^2 can not cancel the leading order term any more. This can be seen from panel (c) where cancellation is not possible for lower m_3 region. However when

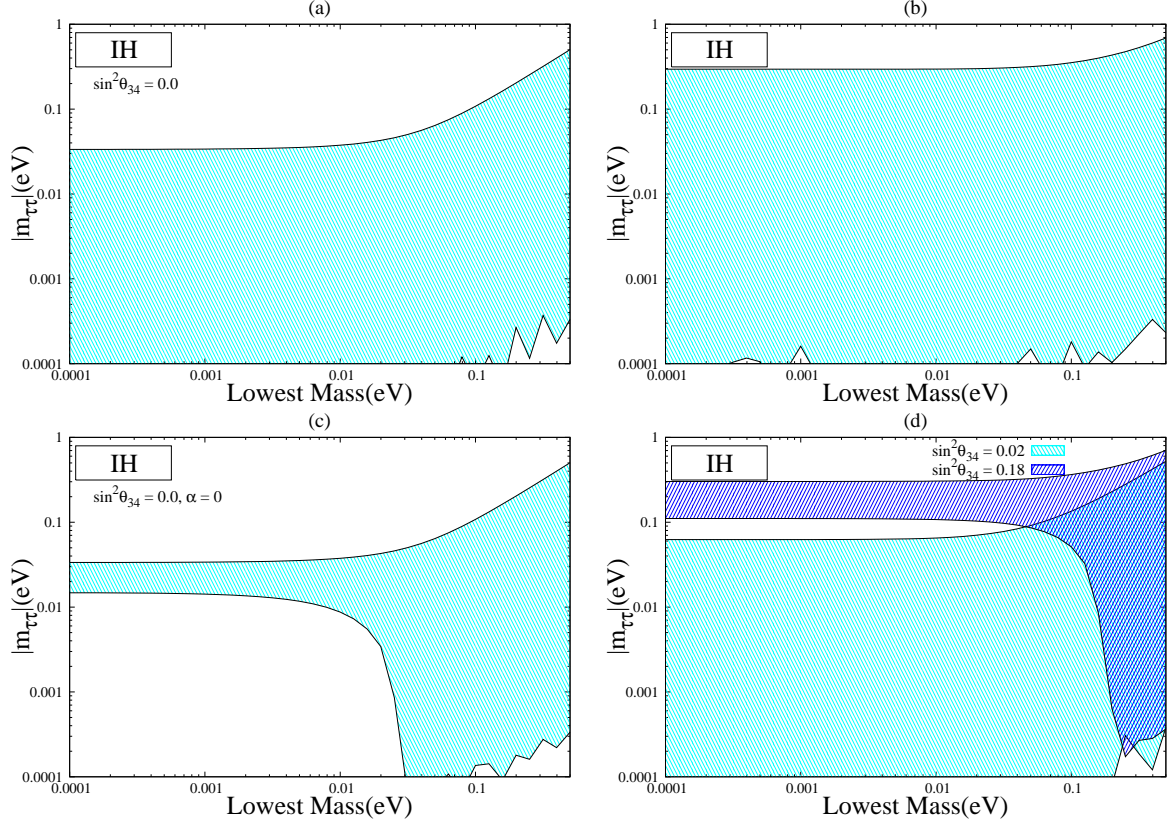


Figure 13: Plots of vanishing $m_{\tau\tau}$ for inverted hierarchy with lowest mass m_3 . Panel (a) correspond to three generation case. In panel (b) all the mixing angles are varied in their full allowed range of parameters (3+1). Panel (c) and (d) are for some specific values of θ_{34} and α .

s_{34}^2 increase to a value of about 0.02 this element can vanish (panel (d) the cyan region). We see that when θ_{34} increase towards its upper bound the λ term becomes large $\mathcal{O}(1)$. Hence, the other terms are not able to cancel this term so we do not get small m_3 region allowed (Panel (d), blue region).

G. The Mass Matrix elements m_{es} , $m_{\mu s}$, $m_{\tau s}$ and m_{ss}

The elements m_{es} , $m_{\mu s}$, $m_{\tau s}$ and m_{ss} are present in the fourth row and fourth column in the neutrino mass matrix. They are the new elements that arises in 3+1 scenario due to the addition of one light sterile neutrino. The expressions for m_{es} and $m_{\mu s}$ are given by

$$\begin{aligned}
m_{es} = & e^{i(2\gamma+\delta_{14})} c_{14} c_{24} c_{34} m_4 s_{14} \\
& + e^{i(2\beta+\delta_{13})} c_{14} m_3 s_{13} \{ -e^{i(\delta_{14}-\delta_{13})} c_{24} c_{34} s_{13} s_{14} + c_{13} (-e^{i\delta_{14}} c_{34} s_{23} s_{24} - c_{23} s_{34}) \} \\
& + c_{12} c_{13} c_{14} m_1 [-s_{12} (-e^{i\delta_{24}} c_{23} c_{34} s_{24} + s_{23} s_{34}) \\
& + c_{12} \{ -e^{i\delta_{14}} c_{13} c_{24} c_{34} s_{14} - e^{i\delta_{13}} s_{13} (-e^{i\delta_{24}} c_{34} s_{23} s_{24} - c_{23} s_{34}) \}] \\
& + e^{2i\alpha} c_{13} c_{14} m_2 s_{12} [c_{12} (-e^{i\delta_{24}} c_{23} c_{34} s_{24} + s_{23} s_{34}) \\
& + s_{12} \{ -e^{i\delta_{14}} c_{13} c_{24} c_{34} s_{14} - e^{i\delta_{13}} s_{13} (-e^{i\delta_{24}} c_{34} s_{23} s_{24} - c_{23} s_{34}) \}].
\end{aligned} \tag{44}$$

$$\begin{aligned}
m_{\mu s} = & e^{i(2\gamma+\delta_{14})} c_{14}^2 c_{24} c_{34} m_4 s_{24} \\
& + e^{i(2\beta+\delta_{13})} m_3 (c_{13} c_{24} s_{23} - e^{i(\delta_{14}-\delta_{13}-\delta_{24})} s_{13} s_{14} s_{24}) \\
& \quad \{ -e^{i(\delta_{14}-\delta_{13})} c_{24} c_{34} s_{13} s_{14} + c_{13} (-e^{i\delta_{24}} c_{34} s_{23} s_{24} - c_{23} s_{34}) \} \\
& + m_1 \{ -c_{23} c_{24} s_{12} + c_{12} (-e^{i\delta_{13}} c_{24} s_{13} s_{23} - e^{i(\delta_{14}-\delta_{24})} c_{13} s_{14} s_{24}) \} \\
& \quad [-s_{12} (-e^{i\delta_{24}} c_{23} c_{34} s_{24} + s_{23} s_{34}) \\
& + c_{12} \{ -e^{i\delta_{14}} c_{13} c_{24} c_{34} s_{14} - e^{i\delta_{13}} s_{13} (-e^{i\delta_{24}} c_{34} s_{23} s_{24} - c_{23} s_{34}) \}] \\
& + e^{2i\alpha} m_2 \{ c_{12} c_{23} c_{24} + s_{12} (-e^{i\delta_{13}} c_{24} s_{13} s_{23} - e^{i(\delta_{14}-\delta_{24})} c_{13} s_{14} s_{24}) \} \\
& \quad [c_{12} (-e^{i\delta_{24}} c_{23} c_{34} s_{24} + s_{23} s_{34}) \\
& + s_{12} \{ -e^{i\delta_{14}} c_{13} c_{24} c_{34} s_{14} - e^{i\delta_{13}} s_{13} (-e^{i\delta_{24}} c_{34} s_{23} s_{24} - c_{23} s_{34}) \}].
\end{aligned} \tag{45}$$

Though the equations seems very complex, one can easily understand the properties of these elements by just looking at the m_4 terms. The m_4 term in m_{es} is proportional to s_{14} . So in general it is quite large $\mathcal{O}(1)$. For this element to become negligible very small values of s_{14}^2 is required. But as this angle is bounded by the SBL experiments, complete cancellations never occurs for both normal and inverted hierarchy (Panel (a), (b) of Fig. 14). Similar predictions are obtained for $m_{\mu s}$ where element cannot vanish as there s_{24}^2 has to be negligible which is not allowed by the data as can be seen from Panel (c), (d) of Fig. 14.

For the element $m_{\tau s}$ the scenario is quite different.

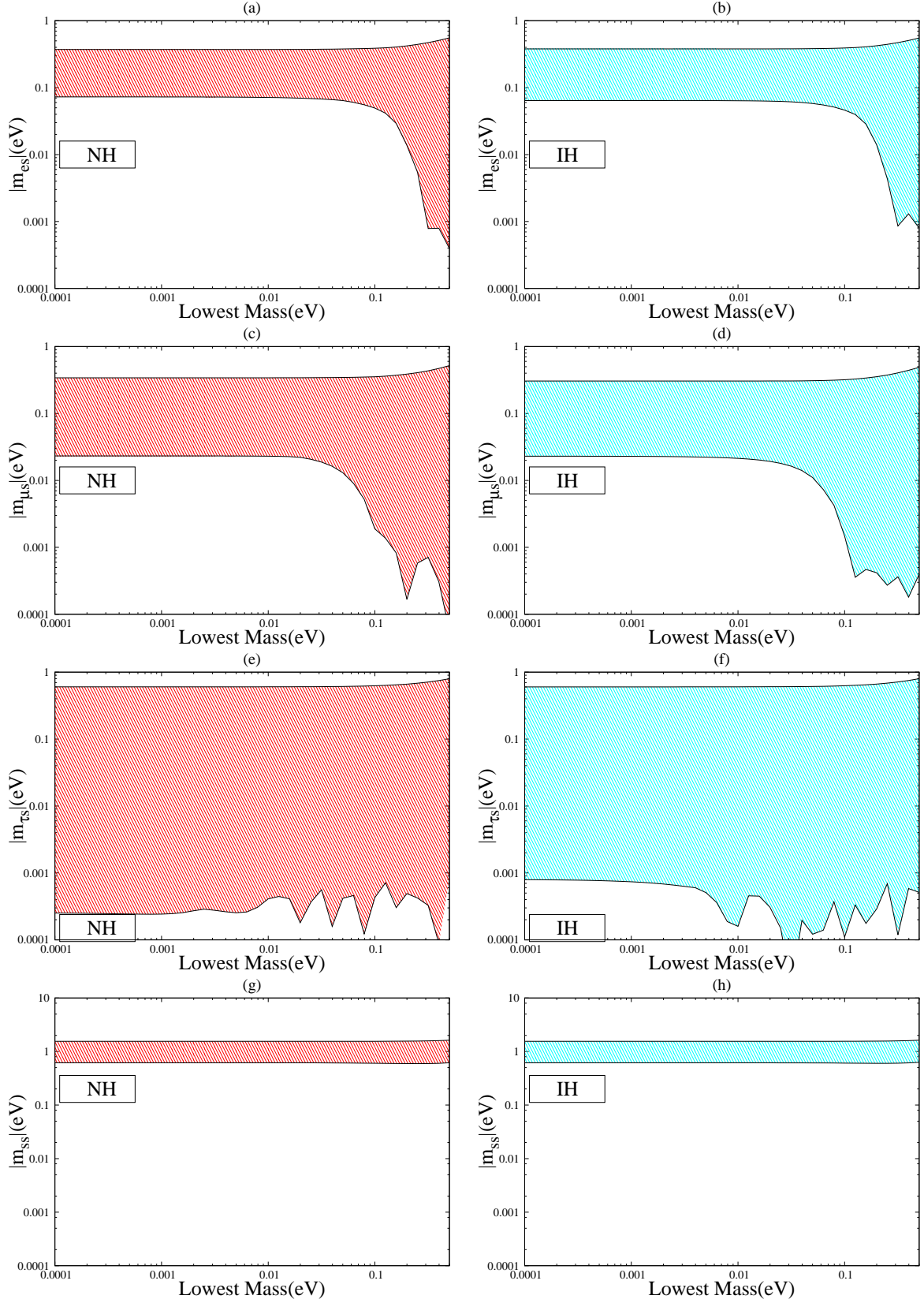


Figure 14: Correlation plots for vanishing $|m_{ks}|$ for both normal and inverted hierarchy. In these plots all the mixing angles are varied in their 3σ allowed range and the CP violating phases are

$$\begin{aligned}
m_{\tau s} = & c_{14}^2 c_{24}^2 c_{34} e^{2i(\delta_{14}+\gamma)} m_4 s_{34} \\
& + e^{2i(\beta+\delta_{13})} m_3 \{ -c_{24} c_{34} e^{i(\delta_{14}-\delta_{13})} s_{13} s_{14} + c_{13} (-c_{23} s_{34} - c_{34} e^{i\delta_{24}} s_{23} s_{24}) \} \\
& \{ -c_{24} e^{i(\delta_{14}-\delta_{13})} s_{13} s_{14} s_{34} + c_{13} (c_{23} c_{34} - e^{i\delta_{24}} s_{23} s_{34} s_{24}) \} \\
& + m_1 [-s_{12} (-c_{23} c_{34} e^{i\delta_{24}} s_{24} + s_{23} s_{34}) \\
& + c_{12} \{ -c_{13} c_{24} c_{34} e^{i\delta_{14}} s_{14} - e^{i\delta_{13}} s_{13} (-c_{23} s_{34} - c_{34} e^{i\delta_{24}} s_{23} s_{34}) \}] \\
& [-s_{12} (-c_{34} s_{23} - c_{23} e^{i\delta_{24}} s_{34} s_{24}) \\
& + c_{12} \{ -c_{13} c_{24} e^{i\delta_{14}} s_{14} s_{34} - e^{i\delta_{13}} s_{13} (c_{23} c_{34} - e^{i\delta_{24}} s_{23} s_{34} s_{24}) \}] \\
& + e^{2i\alpha} m_2 [c_{12} (-c_{23} c_{34} e^{i\delta_{24}} s_{34} + s_{23} s_{34}) \\
& + s_{12} \{ -c_{13} c_{24} c_{34} e^{i\delta_{14}} s_{14} - e^{i\delta_{13}} s_{13} (-c_{23} s_{34} - c_{34} e^{i\delta_{24}} s_{23} s_{24}) \}] \\
& [c_{12} (-c_{34} s_{23} - c_{23} e^{i\delta_{24}} s_{34} s_{24}) \\
& + s_{12} \{ -c_{13} c_{24} e^{i\delta_{14}} s_{14} s_{34} - e^{i\delta_{13}} s_{13} (c_{23} c_{34} - e^{i\delta_{24}} s_{23} s_{34} s_{14}) \}].
\end{aligned} \tag{46}$$

In this case the m_4 term is proportional to θ_{34} and there is no lower bound of it from the SBL experiments i.e. it can approach smaller values. As a result the term with m_4 can be very small. Thus this matrix element can possibly vanish in both hierarchies for whole range of the lowest mass (Panel (e), (f)).

The (4,4) element of the neutrino mass matrix is given as

$$\begin{aligned}
m_{ss} = & c_{14}^2 c_{24}^2 c_{34}^2 e^{2i(\gamma+\delta_{14})} m_4 \\
& + e^{2i(\beta+\delta_{13})} m_3 \{ -c_{24} c_{34} e^{i(\delta_{14}-\delta_{13})} s_{13} s_{14} + c_{13} (-c_{23} s_{34} - c_{34} e^{i\delta_{24}} s_{23} s_{24}) \}^2 \\
& + m_1 [-s_{12} (-c_{23} c_{34} e^{i\delta_{24}} s_{24} + s_{23} s_{34}) \\
& + c_{12} \{ -c_{13} c_{24} c_{34} e^{i\delta_{14}} s_{14} - e^{i\delta_{13}} s_{13} (-c_{23} s_{34} - c_{34} e^{i\delta_{24}} s_{23} s_{24}) \}]^2 \\
& + e^{2i\alpha} m_2 [c_{12} (-c_{23} c_{34} e^{i\delta_{24}} s_{24} + s_{23} s_{34}) \\
& + s_{12} \{ -c_{13} c_{24} c_{34} e^{i\delta_{14}} s_{14} - e^{i\delta_{13}} s_{13} (-c_{23} s_{34} - c_{34} e^{i\delta_{24}} s_{23} s_{34}) \}]^2
\end{aligned} \tag{47}$$

The m_4 term for m_{ss} is proportional to $c_{14}^2 c_{24}^2 c_{34}^2$. One can see that this term is of order one as a result this element can never vanish as is evident from panel (g, h).

IV. CONCLUSIONS

In this paper we analyze systematically the one-zero textures of the 4×4 mass matrix in presence of a sterile neutrino. Assuming neutrinos to be Majorana particles, this is a

symmetric matrix with 10 independent entries. We analyze if the current constraints on oscillation parameters allow each of these to assume a vanishing value. We also study the implications and correlations among the parameters when each matrix element is zero. We expand the mass matrix element in terms of a parameter λ with suitable coefficients χ_{14}, χ_{24} and χ_{34} corresponding to the mixing angles θ_{13} , θ_{14} and θ_{24} . This is motivated by the observation that these angles are of order ~ 0.02 . These expressions facilitate the analytic understanding of the numerical results presented in the different plots. We study the vanishing condition as a function of the lowest mass m_1 (NH) or m_3 (IH) by varying the lightest mass in the range 0.0001 - 0.5 eV.

We find that $|m_{ee}| = 0$ is possible for NH only for higher values of the smallest mass m_1 while for IH it is possible even for lower values. This is in sharp contrast with the 3 generation case where complete cancellation can never take place for IH.

$|m_{e\mu}|$ can vanish over the whole range of the smallest mass for both 3 and 4 neutrino scenarios. However for larger values of the mixing angle s_{24}^2 cancellation is not achieved for smaller m_1 for NH. For IH the cancellation condition depend on the Majorana phase α and the mixing angle θ_{24} . We obtain the correlations between these two parameters required for making this element vanishingly small.

Cancellation is achieved for the element $m_{e\tau}$ for the full range of the lowest mass in the 3+1 scenario. The element $m_{e\mu}$ is related to the element $m_{e\tau}$ by $\mu - \tau$ symmetry. However unlike three generation case θ_{23} in these textures are not related simply by $\bar{\theta}_{23} = (\pi/2 - \theta_{23})$. The mixing angles θ_{24} and θ_{34} are also different in these two textures in general. However for small values of θ_{24} we get $\bar{\theta}_{24} = \theta_{34}$ in these textures. Consequently the role played by θ_{24} for $m_{e\mu}$ is played by θ_{34} in this limit. Thus in this case cancellation is not achieved for larger values of s_{34}^2 in the hierarchical regime for NH. For IH we obtain correlations between α and $\sin^2 \theta_{34}$ for fulfilling the condition for cancellations.

The elements $m_{\mu\mu}$ and $m_{\tau\tau}$ are related by $\mu - \tau$ symmetry. For these cases, cancellation is not possible in the hierarchical zone for IH in the 3 generation case. However the extra contribution coming from the sterile part helps in achieving cancellation in this region. For IH one can obtain correlations between the Majorana phase α and the mixing angle $\theta_{24}(\theta_{34})$ for $|m_{\mu\mu}| = 0(|m_{\tau\tau}| = 0)$.

For $m_{\mu\tau}$ element cancellation was possible for three generation case only for higher values of the lightest mass. However if one includes the sterile neutrino then this element can

vanish over the whole range of the lightest neutrino mass considered.

With the current constraints on sterile parameters it is not possible to obtain $m_{ss} = 0$ while m_{es} and $m_{\mu s}$ can only vanish in the QD regime of the active neutrinos. On the other hand $m_{\tau s}$ can assume vanishingly small values for the lowest mass in the range 0.0001- 0.5 eV, since the angle θ_{34} has only an upper bound from current data and hence can be very small.

The above results can be useful for building models for light sterile neutrinos and shed light on the underlying new physics if future experiments and analyses reconfirm the explanation of the present anomalies in terms of sterile neutrinos.

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